Free Vibration and Buckling Analyses of Functionally Graded Plates Reinforced by Graphene Platelets

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**Abstract.** A new nanocomposite is proposed and evaluated via the free vibration and buckling analyses of plate structures. These nanocomposite plates are manufactured by incorporating graphene platelets (GPLs) into a conventional functionally graded matrix, with the aim of enhancing their overall stiffness. In the present study, the matrix phase is graded along the thickness direction according to the power-law distribution of the constituents, whereas various GPL dispersion patterns along the thickness direction are considered. Based on the novel four-unknown refined plate theory and Hamilton’s principle, the governing equations of motion of the plate are developed. The Navier-type solution scheme is then adopted to obtain the natural frequency and critical buckling load of the nanocomposite plate. Finally, a selected set of results is reported to evaluate the performance of this novel nanocomposite model, and a parametric study is also conducted.

**Keywords:** Free vibration, Buckling, Functionally graded plates, Graphene platelets, Four-unknown refined theory.

1. Introduction

Typical functionally graded materials (FGMs) are microscopically inhomogeneous composite materials with continuous variation of material properties, designed to exhibit specific properties in one or more specific directions [1, 2]. Due to their unique characteristics compared to conventional materials (i.e., high toughness, high strength, good corrosion resistance, and high-temperature resistance capability), FGMs have attracted significant attention in recent decades. The FGMs, nowadays, are applied in many fields such as nuclear engineering, aerospace, automobile, medical area, energy, and defense. Free-vibration and buckling analysis of plate structures are essential for design and practical applications of FGMs. In the existing literature, many studies on the free vibration and buckling analyses of functionally graded (FG) plate structures have been documented [3-11].

 In past years, researchers in the field of nanotechnology have made a breakthrough by developing nanocomposites, which are composite materials comprising polymers, ceramics, or metals reinforced with carbon nanomaterials, to attain exceptional material properties. Among carbon nanomaterials, carbon nanotubes and graphene have gathered significant attention from researchers due to their remarkable mechanical properties. The exceptional high elastic modulus and tensile strength, coupled with its large specific surface area (larger than carbon nanotubes), low cost, and potential for mass production, make graphene or graphene platelets (GPLs) a highly promising candidate to use as a reinforcement material in composites [12-15]. The integration of graphene into metal matrix composites has resulted in noticeable increases in hardness, yield strength, elastic modulus, and in some cases, even ductility. Moreover, the addition of graphene to ceramic matrix composites has continuously improved both toughness and flexural strength [16-19]. Meanwhile, incorporating GPLs into the combination of the metal-ceramic matrix is yet to be studied. Thus, the FGM reinforced by GPLs is proposed and studied in the present study.

So far, many plate theories have been proposed in simulations [11, 20]. For example, the classical plate theory (CPT), first-order shear deformation plate theory (FSDT), and higher-order shear deformation plate theories (HSDTs), [21-25], have been developed and extensively used. The CPT is based on the Kirchhoff hypothesis and appropriate for simulating thin plates. However, it is not adequate for moderately thick and thick plates, as these structures require consideration of shear effects. The FSDT was developed to account for the transverse shear effects. However, the transverse shear strain in the FSDT is assumed to be constant in the thickness direction, thus, a shear correction factor (SCF) must be used. The SCF depends on the geometry, material properties, and boundary conditions of each problem, making it a problem-dependent parameter and not convenient to be determined. As a result, higher-order shear deformation theories (HSDTs) have been established as a means of avoiding the need for the SCF based on the assumption of higher-order variations of the in-plane displacements through the plate thickness; they provide more accurate results. Among HSDTs, the four-unknown refined theory (RPT4) is a good selection in terms of simplicity and computational efficiency [26-28]. The RPT4 involves only four variables but can describe the parabolic distribution of the shear deformation along the plate thickness without the SCF. Many recent works have been conducted to study the vibration, buckling and static behaviors of plate structures using the RPT4 [7, 25-38]. Hence, the RPT4 is adopted for analyzing the plate structures in this study.

 In the present work, the performance of the FG matrix reinforced by GPLs is evaluated by investigating the free vibration and buckling of the plates. Three different patterns of GPL dispersions (i.e., symmetric, asymmetric, and uniform) are considered. The governing equations are based on the RPT4 theory and derived by using the Hamilton’s principle, while the Navier approach is used to find the solution. The effects of GPL weight fraction, GPL distribution pattern, power-law index, and geometric parameters on the natural frequencies of the plate are also provided.

1. Theoretical formulations
	1. Plate model

As shown in Fig. 1, a rectangular nanocomposite plate with length , width , and uniform thickness , is considered. The nanocomposite plate is made of three distinct material phases which consist of two constituents (i.e., ceramic and metal) of the matrix and one phase of the GPL nanofiller. The upper surface of the plate contains a higher concentration of ceramic, while the lower surface of the plate contains a higher concentration of metal.

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**Fig. 1.** A configuration of the nanocomposite plate.

* 1. Material model

For the FG matrix, the volume fractions of component materials denoted by and are assumed to vary continuously through the plate thickness by a power law as follows:

 ()

Then the material properties, such as Young’s modulus and the mass density of the matrix homogenized by the rule of mixture, are written as

 ()

 ()

where the subscripts “1” and “2” denote the two components (i.e., ceramic and metal, respectively) of the FG matrix and is the power law index characterizing the distribution of the volume fraction of the constituent materials.

As shown in Fig. 2, the dispersion of GPLs can be assumed into three distinct patterns: symmetric pattern (pattern A), asymmetric pattern (pattern B), and uniform pattern (pattern C). The volume fraction of GPLs assumed to vary along the z-axis smoothly, can be introduced as

 ()

where , , and are the maximum values of the volume fraction of pattern A, pattern B, and pattern C, respectively. The total amount of GPLs remains constant for all patterns and is characterized by the total weight fraction of GPLs (.

|  |
| --- |
|  |
| (a) Pattern A | (b) Pattern B | (c) Pattern C |

**Fig. 2.** GPLs dispersion patterns.

According to the total weight fraction of GPLs for the whole plate, the maximum values of the GPL volume fraction ,, can be determined as

 ()

in which represent the type of GPL dispersion patterns , , and , respectively, and parameters ,, can be calculated by

 ()

with denoting the mass density of GPLs. The Poisson’s ratio of the nanocomposite is assumed to be constant in the present study and the mass density of the plate is given by the rule of mixture (ROM) as

 ()

where is the volume fraction of the FG matrix. The Young’s modulus of the composite with randomly oriented fillers can be determined by [39, 40]

 ()

where the longitudinal modulus and transverse modulus of the nanocomposite can be computed from two widely used models, the rule of mixture (ROM) and Halpin-Tsai model as described below.

1. Rule of the mixture (ROM)

The longitudinal modulus is given by the ROM, while the transverse modulus is determined by the inverse rule of the mixture as [40, 41]:

  ()

1. Halpin-Tsai model

The longitudinal modulus , and the transverse modulus are calculated from [40, 41]

 ()

where and are defined as

 ()

 (12)

with , , and denoting the average length, width, and thickness of the GPLs, respectively.

As shown in Fig. 3, although there are some minor differences between the two methods, it is evident that they behave similarly in terms of the trend for each dispersion pattern.

* 1. Governing Equations

The displacement field of the FG nanocomposite plate using the four-unknown refined plate theory is expressed as [35]

 ()

 ()

 ()

where and are the displacement components in the , and directions, respectively; and are the mid-plane displacements of the plate in the and directions, respectively; and denote the bending and shear components of the transverse displacement , respectively; and the shape function describing the transverse shear deformation is chosen to be .

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**Fig. 3.** Young’s modulus of FG matrix reinforced by GPLs (weight fraction of 1%) with different dispersion patterns computed by the ROM and the Halpin-Tsai model.

The strain-displacement relations are taken of the form

 ()

where and

 ()

The linear constitutive relations can be expressed as follows:

 ()

In the present study, the equations of motion are derived using the Hamilton's principle as follows [42]:

 ()

where is the strain energy, is the potential energy, and is the kinetic energy of the plate. The first variation of the strain energy can be calculated by

 (20)

where and denote the stress resultants defined as

 ()

The first variation of the potential energy can be obtained as

 , ()

where are in-plane applied loads, respectively. The variation of the kinetic energy can also be written as:

 ()

where

  ()

By substituting Eqs. (20)-(23) into Eq. (19), performing integration by parts, and then enforcing the arbitrariness of , , , and , the equations of motion are obtained as follows:

 ()

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 ()

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where

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The final expression of stress resultants can be obtained as

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where and are defined as

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By substituting the stress resultants into Eqs. (25)-(28), the equations of motion can be expressed in terms of displacement components as follows:

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1. Solution Procedure

In this study, the Navier solution is adopted to obtain the results of the free vibration and buckling of a rectangular nanocomposite plate. For the buckling analysis, the plate is subjected to the in-plane forces in two directions (). Based on the Navier approach, the displacements are chosen to take the following forms to automatically satisfy the simply supported boundary conditions of the plate [5]:

 ()

where are unknown coefficients need to be determined, is the natural angular frequency of the vibration, and .

By substituting Eq. (39) into Eqs. (35) – (38), it results in the following homogeneous system of linear algebraic equations:

 ()

in which

 ()

By neglecting the parameter which contains the in-plane force and solving the eigenvalue problem Eq. (40), the natural frequency of the plate can be obtained. The solutions for the buckling problem can be obtained by neglecting the vibration frequency and solving for critical buckling load .

1. Results and discussion

For the numerical study, the FG plate reinforced by GPLs with the following material properties is considered:

 ()

The Poisson’s ratio is taken as . Note that for results reported further below, the effective material properties of the nanocomposite plate are computed following the ROM.

* 1. Free vibration

Otherwise stated, the normalized natural frequency of the FG plate is defined by

 ()

Since the study on FG nanocomposite plate reinforced by GPLs is yet available in the open literature, the FG plate without GPLs is considered to verify the present results. As shown in Table 1, the results of the present study exhibit a remarkable concurrence with those of Thai and Vo [5] for all cases considered.

**Table 1.** Normalized fundamental frequencies of FG square plate.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Model | Power law index  |  |  |
| 0 | 0.5 | 1 | 4 | 10 |
| 5 | Ref. [5] | 0.2113 | 0.1807 | 0.1631 | 0.1377 | 0.1300 |
|  | Present | 0.2113 | 0.1807 | 0.1631 | 0.1377 | 0.1300 |
| 10 | Ref. [5] | 0.0577 | 0.0490 | 0.0442 | 0.0381 | 0.0364 |
|  | Present | 0.0577 | 0.0490 | 0.0442 | 0.0381 | 0.0364 |
| 20 | Ref. [5] | 0.0148 | 0.0125 | 0.0113 | 0.0098 | 0.0094 |
|  | Present | 0.0148 | 0.0125 | 0.0113 | 0.0098 | 0.0094 |

Next, the FG plate reinforced by GPLs is analyzed. Table 2 shows the normalized natural frequencies of the FG plate without and with GPLs for three different distribution patterns (i.e., patterns A, B and C). The relative changes of natural frequencies in each pattern are also provided. It is apparent that the natural frequencies of the nanocomposite plate increase significantly, about 21.7%-30.8% for the pattern A, about 15.1%-15.6% for the pattern B, about 19.7%-20.8% for the pattern C. Note that the plate with the pattern A produces the highest natural frequency, whereas the plate with the pattern B provides the lowest result. Moreover, the natural frequency of the nanocomposite plate significantly decreases when the value of the ratio increases, as shown in Table 2.

**Table 2.** Comparison of normalized natural frequencies of the square FG plate with and without GPL reinforcement .

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Mode  | FG plate | Pattern A | Pattern B | Pattern C |
| 5 | 1,1 | 0.16312 | 0.20999 | 0.18778 | 0.19724 |
|  |  |  | (28.7%) | (15.1%) | (20.9%) |
|  | 2,1 | 0.36066 | 0.455 | 0.4154 | 0.43461 |
|  |  |  | (26.2%) | (15.2%) | (20.5%) |
|  | 2,2 | 0.52542 | 0.65446 | 0.60561 | 0.63169 |
|  |  |  | (24.6%) | (15.3%) | (20.2%) |
|  | 3,1 | 0.62325 | 0.77143 | 0.71875 | 0.74842 |
|  |  |  | (23.8%) | (15.3%) | (20.1%) |
|  | 3,2 | 0.75695 | 0.92993 | 0.87361 | 0.90763 |
|  |  |  | (22.9%) | (15.4%) | (19.9%) |
|  | 3,3 | 0.95443 | 1.16187 | 1.10288 | 1.14224 |
|  |  |  | (21.7%) | (15.6%) | (19.7%) |
| 10 | 1,1 | 0.04419 | 0.05782 | 0.05087 | 0.05358 |
|  |  |  | (30.8%) | (15.1%) | (21.2%) |
|  | 2,1 | 0.1059 | 0.13733 | 0.1219 | 0.1282 |
|  |  |  | (29.7%) | (15.1%) | (21.1%) |
|  | 2,2 | 0.16312 | 0.20999 | 0.18778 | 0.19724 |
|  |  |  | (28.7%) | (15.1%) | (20.9%) |
|  | 3,1 | 0.19919 | 0.25535 | 0.22932 | 0.24069 |
|  |  |  | (28.2%) | (15.1%) | (20.8%) |
|  | 3,2 | 0.25065 | 0.31952 | 0.28859 | 0.30258 |
|  |  |  | (27.5%) | (15.1%) | (20.7%) |
|  | 3,3 | 0.33049 | 0.41805 | 0.38061 | 0.39844 |
|  |  |  | (26.5%) | (15.2%) | (20.6%) |

Fig. 4 shows the effects of the GPL weight fraction on the relative change of the natural frequency of the nanocomposite plate for . It is seen that the relative frequency is increased by increasing the amount of GPLs. The effect of the aspect ratio on the normalized natural frequency of the plate for the case of is also illustrated in Fig. 5. It is observed that the natural frequency decreases when the aspect ratio or the power law index increases. As a benchmark solution, the normalized natural frequencies of the nanocomposite plate for various aspect ratio , thickness ratio and power law index , with the GPL weight fraction of 5% and dispersion pattern A, is provided in Table 3.

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**Fig. 4.** Relative natural frequency change of each pattern .

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**Fig. 5.** Effect of the aspect ratio on normalized natural frequency of the plate

**Table 3.** Nornalized natural frequencies of the nanocomposite plate with various aspect ratio, thickness ratio and power law index .

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | Power law index  |  |  |  |
| 0 | 0.5 | 1 | 2 | 5 | 10 |
| 0.5 | 5 | 0.5480 | 0.4893 | 0.4549 | 0.4196 | 0.3810 | 0.3631 |
|  | 10 | 0.1658 | 0.1473 | 0.1373 | 0.1287 | 0.1213 | 0.1170 |
|  | 20 | 0.0443 | 0.0392 | 0.0366 | 0.0346 | 0.0332 | 0.0322 |
| 1 | 5 | 0.2533 | 0.2253 | 0.2099 | 0.1959 | 0.1827 | 0.1756 |
|  | 10 | 0.0698 | 0.0619 | 0.0578 | 0.0545 | 0.0521 | 0.0505 |
|  | 20 | 0.0180 | 0.0159 | 0.0149 | 0.0141 | 0.0136 | 0.0132 |
| 3 | 5 | 0.1487 | 0.1320 | 0.1232 | 0.1156 | 0.1092 | 0.1053 |
|  | 10 | 0.0395 | 0.0350 | 0.0327 | 0.0309 | 0.0296 | 0.0288 |
|  | 20 | 0.0100 | 0.0089 | 0.0083 | 0.0079 | 0.0076 | 0.0074 |

* 1. Buckling analysis

For results reported below, the normalized critical bucking load is defined as

 ()

The first comparison, to verify the solution, is carried out for a simply supported plate subjected to different types of in-plane loading as shown in Table 4. It is seen that the results of the present work are identical to those reported by Thai and Choi [43].

**Table 4.** Normalized critical buckling load of a simply supported plate .

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Model |  |  |  |  |  |
| 5 | 10 | 20 | 50 | 100 |
| (-1, 0) | Ref. [43] | 3.4164 | 3.7111 | 3.7930 | 3.8166 | 3.8200 |
|  | Present | 3.4164 | 3.7111 | 3.7930 | 3.8166 | 3.8200 |
| (-1, -1) | Ref. [43] | 2.7331 | 2.9689 | 3.0344 | 3.0533 | 3.0560 |
|  | Present | 2.7331 | 2.9689 | 3.0344 | 3.0533 | 3.0560 |
| (-1, 1) | Ref. [43] | 4.5551 | 4.9481 | 5.0574 | 5.0888 | 5.0934 |
|  | Present | 4.5551 | 4.9481 | 5.0574 | 5.0888 | 5.0934 |

Next, the critical buckling load of the square FG plate reinforced by GPLs is investigated. Table 5 shows the normalized critical buckling loads of the square FG plate with and without GPL reinforcement. It is evident that the critical buckling load of the plate increases significantly, about 48.7%-56.8% for the pattern A, about 18.9%-19.6% for the pattern B, and about 32.2%-33.4% for the pattern C. Moreover, the critical buckling load of the plate decreases when the ratio increases, as illustrated in Table 5.

**Table 5.** Comparison of normalized critical buckling loads of the square FG plate with and without GPL reinforcement. .

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | a/h | FG plate | Pattern A | Pattern B | Pattern C |
| (-1, 0) | 5 | 8.2245 | 12.2333 | 9.8352 | 10.8722 |
|  |  |  | (48.7%) | (19.6%) | (32.2%) |
|  | 10 | 9.3391 | 14.4660 | 11.1156 | 12.4360 |
|  |  |  | (54.9%) | (19.0%) | (33.2%) |
|  | 20 | 9.6675 | 15.1598 | 11.4906 | 12.9011 |
|  |  |  | (56.8%) | (18.9%) | (33.4%) |
| (-1, -1) | 5 | 4.1122 | 6.1166 | 4.9176 | 5.4361 |
|  |  |  | (48.7%) | (19.6%) | (32.2%) |
|  | 10 | 4.6696 | 7.2330 | 5.5578 | 6.2180 |
|  |  |  | (54.9%) | (19.0%) | (33.2%) |
|  | 20 | 4.8337 | 7.5799 | 5.7453 | 6.4505 |
|  |  |  | (56.8%) | (18.9%) | (33.4%) |

The effects of the GPL weight fraction on the relative change of the critical buckling load of the nanocomposite plate subjected to both uniaxial compression and biaxial compression are reported in Fig. 6 and Fig. 7, respectively. It is observed that the critical buckling loads of the nanocomposite plate are significantly improved compared to those of the FG plate without GPL reinforcement. In addition, the nanocomposite plate with the GPL dispersion pattern A provides the largest buckling load among the three patterns in both cases of the in-plane forces. As a benchmark solution, Table 6 presents the normalized critical buckling loads of the nanocomposite plate for various aspect ratios , thickness ratios and power law indexes , where the GPL dispersion pattern A is considered. We can also see that the critical buckling load decreases when the power law index (*N*) increases, as shown in Table 6.

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**Fig. 6.** Effect of the GPL weight fraction on the relative change of critical buckling load of the nanocomposite plate subjected to uniaxial compression .

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**Fig. 7.** Effect of the GPL weight fraction on the relative change of critical buckling load of the nanocomposite plate subjected to biaxial compression .

**Table 6.** Normalized buckling loads of the nanocomposite plate with various aspect ratios, thickness ratios, and power law indexes .

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | Power law index  |  |  |  |
| 0 | 0.5 | 1 | 2 | 5 | 10 |
| 0.5 | 5 | 19.2772 | 14.1863 | 11.7048 | 9.4389 | 7.3080 | 6.4314 |
|  | 10 | 27.5124 | 19.9772 | 16.5686 | 13.8272 | 11.6197 | 10.5140 |
|  | 20 | 30.8291 | 22.2657 | 18.5059 | 15.6638 | 13.6553 | 12.5233 |
| 1 | 5 | 10.1350 | 7.3852 | 6.1166 | 5.0580 | 4.1543 | 3.7289 |
|  | 10 | 12.0410 | 8.7066 | 7.2330 | 6.1032 | 5.2769 | 4.8246 |
|  | 20 | 12.6368 | 9.1155 | 7.5799 | 6.4369 | 5.6610 | 5.2089 |
| 3 | 5 | 6.2128 | 4.5079 | 3.7398 | 3.1270 | 2.6404 | 2.3932 |
|  | 10 | 6.8817 | 4.9690 | 4.1303 | 3.4981 | 3.0544 | 2.8028 |
|  | 20 | 7.0723 | 5.0997 | 4.2412 | 3.6053 | 3.1793 | 2.9285 |

1. Conclusion

In the present study, the free vibration and elastic buckling of FG plates reinforced by GPLs using the four-unknown refined plate theory have been investigated for the first time. The Hamilton’s principle has been utilized to derive the equations of motion and the Navier’s solution has been implemented to obtain the solutions for the natural frequencies and critical buckling loads. Results from the present study lead to the following conclusions:

1. The natural frequency and critical buckling load of the FG plate reinforced by GPLs are significantly improved by increasing the GPL weight fraction. In particular, the increase is about 15.1%-30.8% for the free vibration problem (*b/a = 1,* ), and about 18.9%-56.8% for the buckling problem (*a/h = 10,* ) in investigated cases.
2. The highest natural frequency and critical buckling load of the plate could be found in the GPL dispersion pattern A. Therefore, the pattern A is found the most effective dispersion pattern in comparison with the other two cases considered.
3. As expected, the natural frequency and critical buckling load decrease when the length-to-thickness ratio and the power law index (*N)* increases.

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