Mixed Finite Element Formulation for the Nonlinear Analysis of Buckling Behavior of Truss under Effects of Temperature Variation

Vu Thi Bich Quyen 1, Dao Ngoc Tien 1, Truong Manh Khuyen 1

1 Faculty of Civil Engineering, Hanoi Architectural University, Vietnam

Email: bquyen1312@gmail.com

**Abstract**. This paper proposes a novel approach to consider the effects of temperature variability in the nonlinear finite element analysis of buckling behavior of truss system subjected to initial mechanical load. Generally, using displacement-based finite formulation the procedure for solving geometrically nonlinear problem of the truss system subjected to temperature load is required to implement the temperature deformation constraint depending on the incremental element length to the master stiffness equation. For escaping the mathematical difficulties of treating the additional constraint of temperature deformation this research proposes to formulate this problem based on mixed finite element formulation. The balanced mixed equation of truss considering temperature loading effect is constructed based on the principle virtual work with establishing the mixed matrix of truss element, which consists the element temperature deformation, considering large displacement. For investigating the effects of temperature variation to the nonlinear buckling behavior of truss this research develops the incremental-iterative algorithm for solving the nonlinear equation with incremental displacement and temperature by using Newton-Raphson technique. Based on proposed algorithm the calculation program is written for numerical investing the effects of temperature variation the buckling behavior of plan truss.

**Keywords:** Nonlinear analysis of truss, Buckling behavior, Mixed finite formulation, Temperature variation, Temperature load, Large displacement.

1. Introduction

In many practical cases, the buckling behavior of the truss needs to be considered for design purposes when the truss is subjected to temperature load. However, the problem of elastic buckling of truss subjected to the mechanical temperature load is substantially different. In fact, not as many research works have been published regarding temperature load variation. Especially, little study for nonlinear analysis of the buckling behavior of trusses under temperature variation has been found [1,2]. In geometrical nonlinear finite element analysis, the temperature deformation of truss element is dependent on the incremental element length. In geometrical linear finite element analysis, the temperature loads usually are calculated using the restraining method suggested by J. M. C. Duhamel (1838) by adding the thermal loads to the nodal external force vector as equivalent loads [3-5]. Generally, in geometrical nonlinear finite element analysis based on displacement-based formulation, solving a geometrically nonlinear problem of the system under thermal loading is required to implement the temperature deformation constraint depending on the incremental element length to the master stiffness equation by using mathematical methods for constrained optimization such as penalty augmentation method or Lagrange multiplier adjunction method [6-9]. The implementation of temperature deformation constraint considerably increases the difficulty in finite element formulation of nonlinear buckling analysis of truss. Based on mixed finite formulation, this research introduces novel approach to formulate the geometrical nonlinear buckling problem of truss under the effects of temperature load without difficulties of imposing the temperature deformation constraint. The mixed matrix of truss element, consisting element temperature deformation, is constructed based on large displacement assumption. Based on the formulated equation this research develops the incremental-iterative algorithm for solving the nonlinear equations with incremental displacement and temperature by using Newton-Raphson technique. Based on proposed algorithm, the calculation program is written for investigating the effects of temperature variation to the nonlinear buckling behavior of truss. The numerical results are presented to verify the efficiency of the proposed method.

1. Problem formulation
	1. *Balanced equation of mixed truss element considering temperature loading effect*

Consider the homogeneous truss bar subjected to uniform thermal load due to a constant temperature change shown in Fig. 1. The axial deformation of truss element is defined as

  (1)

 Where: *L*o and *L* - distances between two end nodes of the truss element before and after deformation due to mechanical load; *LT*– the length of truss element after deformation considering temperature deformation; - the change in length due to temperature deformation [10] in case of temperature change with linear thermal expansion coefficient 

 Using the relationship of node displacements in geometry, the length of the truss element is computed as follows

 (2)

 Where  nodal displacements in a global coordinate system



**Figure 1.** Mixed truss element cosidering thermal loading effect

 Work done by internal forces can be computed by

 (3)

 Where: *N* - internal normal force of truss element;

 The virtual external work can be defined as

 (4)

 Where:  - nodal forces in a global coordinate system.

 The total work done by internal and external forces is obtained by summing equation (3) and equation (4), having

 (5)

 Where  - resultant external force at the ith cross section after deformation.

 Based on the principle of virtual work, in equilibrium the virtual work of the forces applied to a system is zero, getting

 (6)

 Replacing the axial deformation of truss in equation (6) with deformation in equation (1), then expressing axial force through deformation, getting

 (7)

 Combining equation (7) with equation (2), having

  (8)

 Designate 

 The equation (8) can be compactly written as

  (9)

 Where

  is the vector consists of nodal variables (primary unknowns) in the global coordinate system, including displacement unknowns and external force unknown at the ith cross section 

 Based on mixed finite element formulation, the balanced equation truss element can be written as

  (10)

 Designating 

 Input incremental loading into the Eq. (10), getting

 (11)

  is second order infinitesimal and it can be negligible, the equation (11) is becoming

  (12)

 Setting 

 The equation (12) can be compactly written as

 (13)

 In the equation (13), is a mixed matrix of truss element considering the temperature loading effect and large displacement, is given by

 (14)

 Where

 

 

 

 

 

 

 

## Balanced equation of truss system

The global equation of truss system can be established by assembling elements . Based on the equation of (10), the equilibrium condition of the system can be expressed as

 (15)

 Where:  ;

“*m*” is a number of truss elements and “n” is number of unknowns;

 The equation (15) is a nonlinear system, it can be expressed in incremental form as

 (16)

 Setting a global mixed matrix, developed by assembling mixed matrix of truss elements

  (17)

 The equations (16)&(17) can be compactly written as

 (18)

 (19)

 For geometrically nonlinear analysis of system subjected to thermal load increasingly putting temperature parameter  and vector of element deformation due to the thermal load  (the initial value is guessed in solving process using incremental-iterative solution techniques). The initial thermal expansion vector can be expressed as

 (20)

 When the mechanical load remains unchanged, the equation (18) is becoming

 (21)

1. Algorithm for solving problem based on Newton-Raphson technique

Based on Newton-Raphson method, in this research a nonlinear finite element analysis performed using an incremental formulation, in which the variables including displacement and temperature are updated incrementally corresponding to successive load steps in order to trace out the complete solution path (equilibrium path).



**Figure 2.** Newton Raphson technique for solving nonlinear truss problem

Using Newton-Raphson technique [11-13], external loads are computed at the first iteration of each incremental step and held constant throughout the remaining iterations in the step, as illustrated in Fig.2. The following iterative algorithm of jth iterative step at the ith thermal load step is proposed

* Computing the axial deformation of truss element  corresponding temperature parameter : **
* Computing the tangent mixed matrix at the (j-1)th iterative step: 
* Computing the incremental displacement due to the residual load : 
* Computing the :
* Computing corresponding load in equilibrium path: 
* Computing the residual load: 
* Comparing the residual load to convergence tolerance :

+ If  , go to the next thermal load step: ,**;

+ If  is not satisfied, go to next iterative step: 

Based on the proposed iterative algorithm, the block diagram of algorithm for solving the nonlinear system of equations is established (shown in Fig.3).

No

Yes

 

 

 **



 **

 Step: i

Computing global mixed matrix . Setting number of load increments n, ( i=1..n); Setting convergence tolerance ; Setting thermal load step . Setting initial guessed vector ;. Setting initial value (i=1): 

 **

 **

 **

**

**



 **

 **

**Figure 3.** Block diagram of algorithm for solving nonlinear system of equations based on Newton Raphson technique

1. Numerical investigation

Based on proposed incremental-iterative algorithm, the calculation program to solve the above truss problem is written using Matlab software.

## Example formulation

Investigate the nonlinear buckling behavior of truss system subjected to constant mechanical load and varied temperature load ΔT shown in Fig. 4. All of the truss bars made of the same material and have the same cross-sectional area. The parameters are given





**Fig. 4.** Examined system, designating unknowns of the system in the mixed formulation

 The unknowns of the problem are designated as shown in Fig.4, including  - nodal displacement unknowns and  - axial force unknowns.

## Numerical results

The calculating results are temperature load-displacement and temperature load-internal force equilibrium path in two cases of solving, shown in Fig.5-6.



**Fig. 5.** Temperature load-displacement equilibrium path  in cases of temperature variation 



**Figure 6.** Temperature load-internal force equilibrium path  in cases of temperature variation 

## Comments

The temperature variation significantly influence to the nonlinear buckling behavior of truss system. Due to the effects of temperature variation the snap-through behavior is appearing in the equilibrium path. The limit point of temperature variation  can be found in the graphics in Fig.5 and 6.

1. Conclusion

The mixed based formulation mathematical model for solving the geometrically nonlinear buckling problem of truss under temperature loading effects has significant advantage over displacement based formulation model. Taking mixed unknowns gives possibility to insert the length-dependent temperature deformation into the mixed matrix of truss element and to simplify the solving algorithm. The proposed method and established algorithm can be effectively used to investigate the nonlinear buckling and snap-through behaviour of truss, to define the equilibrium path and limit point of temperature variation.

References

1. Adnan Ibrahimbegovic, Emina Hajdo and Samir Dolarevic, Linear instability or buckling problems for mechanical and coupled thermomechanical extreme conditions, Coupled Systems Mechanics, Vol. 2, No. 4, 349-374, 2013.
2. Gebrail B,, Rasim T,, Yusuf C. T., Analyses of Truss Structures under Thermal Effects, Recent Advances in Civil Engineering and Mechanics, 2014.
3. K.J. Bathe, Finite Element Procedures, Prentice Hall, 2016
4. O. C. Zienkiewicz, R. L. Taylor, The Finite Element Method for Solid and Structural Mechanics (7th edn), Butterworth-Heinemann, 2014
5. Mircea Radeş, Finite Element Analysis, Editura Printech, 2006
6. J. J. Strodiot, Numerical Methods in Optimization, Namur - Belgium, 2002
7. W. Sun and Y -X. Yuan, Optimization Theory and Methods - Nonlinear Programming, Springer, 2006.
8. Bertsekas D. P., Constrained optimization and Lagrange multiplier method, Academic Press, Inc., 1982
9. J.T. Oden, Penalty-finite element methods for constrained problems in elasticity, Symposium on finite element method, Hefei politecnical university, China, May 19-23, 1981
10. Naotake Noda, Richard B. Hetnarski, Yoshinobu Tanigawa, Thermal stresses, Taylor & Francis Group, 2003
11. M. A. Crisfield, Nonlinear Finite Element Analysis of Solids and Structures, John Wiley & Sons Ltd., 1997
12. Crisfield, M. A., “A Fast Incremental/Iterative Solution Procedure That Handles Snap-Through,” Comput. Struct., 13(1–3), pp. 55–62., 1981
13. Riks, E., “The Application of Newton’s Method to the Problem of Elastic Stability,” ASME J. Appl. Mech., 39(4), pp. 1060–1066, 1972