Study on an Unscented Kalman filter to identify dynamic parameters of the ball screw feed drive system

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Abstract. Today, the problem of system identification (SI), which includes estimating the dynamic parameters of a complex mechanical system, is an interesting research topic. Previous studies have succeeded in analyzing vibration signals to extract some features to diagnose the condition, which is a special method for identifying ball screw feed drive systems (BFDS). However, these methods require measurement techniques with specialized instruments and complex data processing methods, causing difficulties in their widespread application in practice. This paper presents the application of an unscented Kalman filter (UKF) to estimate the vibrational responses of BFDS. First, a dynamic modeling method for BFDS is proposed to determine dynamic parameters such as mass/inertia, stiffness, and damping. Then, the vibration responses, including displacement, velocity, and acceleration, will be calculated and numerically simulated by the Runge-Kutta 4th order (RK4th). This vibration response is also the input data for estimating states and dynamic parameters using UKF. The combination of the mathematical model and the powerful unscented transformation based on the Kalman filter will allow us to estimate the vibration responses and the dynamic parameters of the system accurately. The feasibility of the UKF method was evaluated by the correlation between the state estimation results and the vibration responses of the RK4th. Besides that, it is possible to evaluate the accuracy of UKF through the error between the data input and the estimated results of the dynamic parameters of BFDS. The preliminary results of this paper demonstrate that the UKF method can be applied to system identification and monitoring of the status of the BFDS system. This approach has the potential to improve the safety and reliability of BFDS systems, as it allows for real-time monitoring and early detection of any potential issues. Further research is needed to fully validate the effectiveness of this method in practical applications.

Keywords: Ball screw feed drive system (BFDS), Dynamic Modeling Method, Runge-Kutta 4^{th} Order (RK4th), Unscented Kalman Filter (UKF), Dynamic Parameters Estimation.

1 Introduction.

Several systems are used to convert the motor's rotary motion into linear motion (LM), such as belt drives, rack and pinion drives, and ball-nut-screw drives (BFDS). In modern milling machines, accuracy is dictated by the high rigidity and efficiency of the drive system. Therefore, the BFDS system is judged to be the most suitable for use in computer numerical control (CNC) machine tools [1]. During operation, the vibration of the BFDS is the main factor affecting the product's quality, performance, and processing time. Mechanical backlash or geometrical error is considered one of the common causes affecting the performance of BFDS. Rotating parts in CNC milling machines are frequently used, especially in operations that convert rotary motion from the motor to translating motion of the working table via BFDS [2]. Due to the continuous operation under high load and high-intensity conditions, the BFDS inevitably wears and fatigues [3], thereby causing a decrease in the stiffness of the transmission system, especially the ball screws. In the last few years, some methods of surveying and assessing the condition based on vibration have been studied and applied to ensure the stable operation of the machine.

In order to investigate and evaluate the operating status of the CNC machine, recent studies have conducted dynamic modeling for the BFDS system by various methods, such as the lumped parameter method, the finite element method, and the *hybrid* method [4]. Based on the study during operation and monitoring the machine's condition before and after the loss of accuracy, several assumptions are made to simplify the dynamic modeling. Based on the study during operation and monitoring the machine's condition before and after the loss of accuracy, several assumptions are made to simplify the dynamic modeling. The lumped parameter method is evaluated as simple for easily modeling a dynamic model based on the generalized coordinates and applying numerical methods to solve it quickly [5]–[7]. Many scientific papers have processed the finite element method (FEM) to consider the dynamic behavior of BSFS in as much detail as possible. This method requires that the dynamic parameters of the system be calculated and determined precisely. The joints and stiffness of the dynamic joints are also considered [8]–[10]. A hybrid modeling approach that uses a combination of lumped method and dynamic parameters to investigate the vibration response of a ball screw, where the screw shaft is considered as a Timoshenko beam model including torsional, axial, and flexural [11]–[15]. This method can comprehensively reflect the vibration states, including free and forced vibrations of BFDS, when the position of the table changes with time. Besides, the dynamic parameters affecting the system vibration can be approximated, helping to solve the vibration problem accurately and quickly.

Nowadays, research on monitoring and predicting the health condition of the machine has enormously important contributions to make in establishing operation and maintenance procedures. The purpose is to improve safety for machining and product manufacturing. It is possible to monitor and predict the condition or failure of the machine based on the data from the vibration signal during operation. The modal analysis includes experimental mode analysis (EMA) and operational mode analysis (OMA), which is an analytical method based on vibration theory to determine the modal parameters of the complex mechanical system as BFDS. This method allows us to determine the system's dynamic parameters, including the eigenvalues, modal frequency, damping ratio, and modal geometry [16]. However, BFDS has a large displacement and fastchanging speed under actual working conditions, which may completely differ from the theoretical conditions. According to the investigation of the ball screw vibration responses [17], it can be considered that there are two main vibration modes: axial mode and rotation mode. That makes it difficult for modal analysis methods to install accelerometers. In addition, the collected measurement signal can only determine the vibration responses in the axial directions. Still, it cannot determine the response of the torsion vibration mode, leading to certain limitations.

Kalman filter (KF) is a tool that scientists have been interested in, researched, and strongly developed in recent years to estimate the state and dynamical parameters of the mechanical system. KF has made important contributions to the application of high-tech fields such as aerospace, civil engineering, and, most recently, industrial engineering [18]. Nowadays, KF has been strongly developed, which can be mentioned as the Extended Kalman Filter (EKF) or the Unscented Kalman Filter (UKF), which are commonly used for the determination of non-linear mechanical systems [19]. In this study, we propose the application of UKF to estimate the vibration response and the dynamic parameters of BFDS under the excitation force. First, BFDS has modeled stiffness, as a mechanical system with finite degrees of freedom, theoretically determining dynamic parameters such as mass/inertia, stiffness, and viscous damping. A hybrid dynamics model can investigate ball screw vibration responses; dynamic parameters such as mass or inertia, and damping are lumped parameter models, while the ball screw and mechanism have equivalent stiffness values. Based on vibration theory, solving the eigenvalue problem allows determining the eigenvalues, natural frequency, vibration frequency, and eigenvector in the modes of BFDS. The Runge-Kutta 4th order method is used with data input as dynamic parameters and excitation force for numerical simulation of vibration responses such as displacement, velocity, and acceleration at generalized coordinates. The response results from RK4th will be input into the UKF state estimation process. Moreover, UKF is superior when estimating dynamic parameters such as mass/inertia, stiffness, and damping. Finally, UFK is used to estimate the dynamic parameters of the BFDS model and compare the results of that estimated state. The results show that the modeling method for BFDS can be evaluated as suitable for the computational-numerical simulation model using RK4th and UKF. Correlations and percentage deviations between the estimated results were evaluated to demonstrate the effectiveness and superiority of the UKF method. The results of this paper will be the premise for further studies to apply the UKF to the problem of identifying systems for BFDS.

2 Mathematic and method.

2.1 Ball screw feed drive system.

Today, BFDS is widely used for machines that need to ensure position accuracy because of its high load-carrying capacity and rigidity. BFDS combines moving components such as a motor, ball screw, bearings, linear motion guideways, and table. In particular, the accuracy of BFDS is mainly determined by the condition of the ball screw, because this is a dynamic load-bearing part that converts the motor's rotation motion into the translating motion. The workpiece load and the cutting force act directly on the machine table, which is transmitted to the linear guide system and the ball screw nut fastened to the table. Two thrust ball bearings are rigidly attached to the base of the machine with the task of supporting - blocking, and only allowing the screw shaft to rotate to transmit torque from the motor. The relationship between the generalized co-ordinates in the dynamic modeling diagram of BFDS is shown in **Fig. 1.**

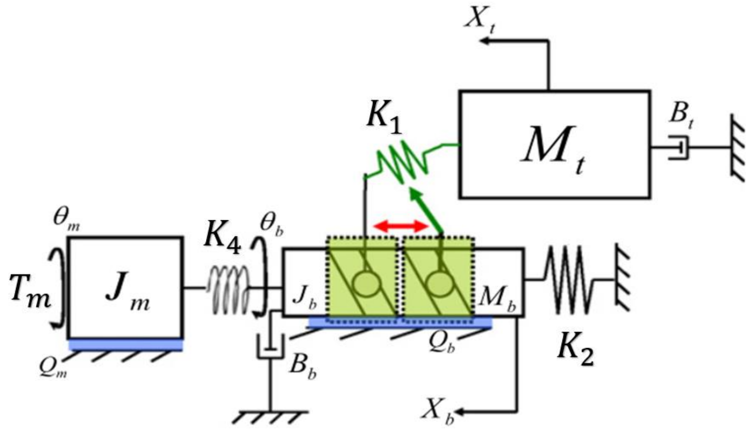

Fig. 1. The BFDS dynamic modelling [5].

In the dynamic model of the proposed BFDS, the dynamic parameters are as follows: mass/inertia, ball screw-ball nut stiffness and damping of the components. In general, the equations of motion are performed as a matrix called Lagrange's equation, which can be represented as follows Eq.(1)

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\} \tag{1}$$

In which, $[M]$ is mass matrix, $[C]$ is damping matrix, $[K]$ is stiffness matrix, and $\{x\}$ is generalized coordinate vector. With a four-degree-of-freedom system (4-dofs) of BFDS shown in Fig.1, the dynamic equations can be expressed in terms of Eq. (2) as follows:

$$\begin{bmatrix} M_t & 0 & 0 & 0 \\ 0 & M_b & 0 & 0 \\ 0 & 0 & J_b & 0 \\ 0 & 0 & 0 & J_m \end{bmatrix}\begin{bmatrix} \ddot{X}_t \\ \ddot{X}_b \\ \ddot{\theta}_b \\ \ddot{\theta}_m \end{bmatrix} + \begin{bmatrix} B_t & 0 & 0 & 0 \\ 0 & B_b & 0 & 0 \\ 0 & 0 & Q_b & 0 \\ 0 & 0 & 0 & Q_m \end{bmatrix}\begin{bmatrix} \dot{X}_t \\ \dot{X}_b \\ \dot{\theta}_b \\ \dot{\theta}_m \end{bmatrix}$$
$$+ \begin{bmatrix} K_1 & -K_1 & -\beta K_1 & 0 \\ -K_1 & K_2 & \beta K_1 & 0 \\ -\beta K_1 & \beta K_1 & K_3 & -K_4 \\ 0 & 0 & -K_4 & K_4 \end{bmatrix}\begin{bmatrix} X_t \\ X_b \\ \theta_b \\ \theta_m \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ T_m \end{bmatrix} \tag{2}$$

The stiffness matrix $[K]$ of the ball screw includes the stiffness of the ball screw nut (K_1) and the stiffness of the screw shaft. The hardness of a ball screw nut (K_1) depends on the degree of preload (P) and can be determined based on the manufacturer's catalogue stiffness. The stiffness of the screw shaft is divided into equivalent axial stiffness (K_2) and equivalent torsional stiffness (K_3) and torsional stiffness of the screw shaft (K_4). The stiffness of the lead screw in this dynamic model depends on the physical geometry and material properties. **Table. 1** shows the dynamic parameters of BFDS.

Table. 1. The dynamic parameters of BFDS

Symbol	Dynamic parameters	Value
M_t	Mass of Working table	15 (kg)
M_b	Mass of the ball screw	2.56 (kg)
J_m	Inertial moment of the motor	1.73×10^{-5} (kg. m^2)
J_b	Inertial moment of the ball screw	1.4475×10^{-4} (kg. m^2)
K_1	The ball screw nut stiffness	1.374×10^{8} (N/m)
K_2	The equivalent axial stiffness of ball screws	1.874×10^{8} (N/m)
K_3	The equivalent torsional stiffness of ball screws	1153.93 (Nm/rad)
K_4	The torsional stiffness of ball screws	717 (Nm/rad)
β	The axial displacement of the ball screw shaft caused by rotation	2.55×10^{-3} (Nm/rad)
B_t	The viscous damping coefficient of the guideway of the working table	1 (Ns/m)
B_b	The viscous damping coefficient of the supporting bearing of the ball screw	1 (Ns/m)
Q_m	The rotational viscous damping coefficient of the motor	0.002 (Nms/rad)
Q_b	The rotational viscous damping coefficient of the support bearing	0.002 (Nms/rad)
T_m	The motor torque	0 (N. m)

It is found that the position of the working table also affects the axial stiffness of the screw shaft (K_2), which means that the values (K_2) of the matrix [K] change depending on the position changing of the working table during operation. Therefore, for the eigenvalue problem, it is generally assumed that no damping or excitation force is acting on the system. So the eigenvalues and eigenvectors will change as the stiffness (K_2) changes. The eigenvalues problem, which Eq. (3) is called the characteristic equation:

$$|[K] - \lambda[M]| = 0 \tag{3}$$

Where $\lambda = \omega^2$, λ defined as the eigenvalue, ω is defined as the angular frequency of the system. The solution to this problem would be to provide four natural frequencies f_n in the four corresponding vibration modes. In the case of BFDS in the stationary state, the dynamic parameter values do not change, and the eigenvalue problem allows quick investigation of values such as natural frequencies. However, when investigating the vibrational response of the system muscle under the action of the excitation force, we need another, more powerful numerical method.

2.2 Numerical method Runge-Kutta 4th order (RK4th)

In a multi-degree-of-freedom mechanical system, the matrix equation of motion eq.(1) is used to describe the acceleration vector that becomes:

$$\{\ddot{x}(t)\} = [M]^{-1}(\{F(t)\} - [C]\{\dot{x}(t)\} - [K]\{x(t)\}) \tag{4}$$

By considering displacement as well as velocity as an unknown variable, a new vector equation is expressed as: $\{X(t)\} = \begin{Bmatrix} x(t) \\ \dot{x}(t) \end{Bmatrix}$ and there is:

$$\{\dot{X}(t)\} = \begin{Bmatrix} \dot{x}(t) \\ \ddot{x}(t) \end{Bmatrix} = \begin{Bmatrix} \dot{x}(t) \\ [M]^{-1}(\{F(t)\} - [C]\{\dot{x}(t)\} - [K]\{x(t)\}) \end{Bmatrix} \tag{5}$$

Equation. (5) can also be rearranged:

$$\{\dot{X}(t)\} = \begin{bmatrix} [0] & [I] \\ [M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} \begin{Bmatrix} x(t) \\ \dot{x}(t) \end{Bmatrix} + \begin{Bmatrix} 0 \\ [M]^{-1}\{F(t)\} \end{Bmatrix} \tag{6}$$

with: $\{\dot{X}(t)\} = f(X,t)$

Which:

$$f(X,t) = [A]X(t) + \tilde{F}(t) \tag{7}$$

$$[A] = \begin{bmatrix} [0] & [I] \\ [M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} \tag{8}$$

$$\tilde{F}(t) = \begin{Bmatrix} 0 \\ [M]^{-1}\{F(t)\} \end{Bmatrix} \tag{9}$$

Based on this, the formula used to estimate $\{X(t)\}$ at different grid points t_i is reproduced with the following equation:

$$\{X(t)\} = \{X_i\} + \frac{1}{6}[\{K_1\} + 2\{K_2\} + 2\{K_3\} + \{K_4\}] \tag{10}$$

In which:

$$\{K_1\} = h \times f(\{X_i\}, t_i) \tag{11}$$

$$\{K_2\} = h \times f\left(\{X_i\} + \frac{1}{2}\{K_1\}, t_i + \frac{1}{2}h\right) \tag{12}$$

$$\{K_3\} = h \times f\left(\{X_i\} + \frac{1}{2}\{K_2\}, t_i + \frac{1}{2}h\right) \tag{13}$$

$$\{K_4\} = h \times f(\{X_i\} + \{K_3\}, t_i) \tag{14}$$

2.3 Estimation method using Unscented Kalman Filter (UKF)

The Unscented Transform (UT) [20] is a method of statistical computation of a random variable undergoing a non-linear transformation. UKF uses UT to create an easier way to approximate a Gaussian random variable (GRV) to avoid approximating an arbitrary density function by a non-linear transformation. The features of UT include: Calculation of the set of Sigma points χ, Each Sigma point will have a weight w, Transform the points through a non-linear function, and Calculate the Gaussian distribution from the weighted points.

Sigma points χ, assuming that the state functions follow a Gaussian distribution, we have the following definitions from [21]:

$$\chi_0 = \bar{x} \tag{15}$$

$$\chi^i = \bar{x} + \sqrt{(n+\lambda)P_i^{XX}},\ i = 1,2,\dots,n \tag{16}$$

$$\chi^i = \bar{x} - \sqrt{(n+\lambda)P_i^{XX}},\ i = n+1, n+2, \dots, 2n \tag{17}$$

In which: P_0^{XX} is covariance error matrix; $\bar{x}$ is states' vector mean; n is the number of states; λ is the scaling parameter; i is the column vector number.

The weight w corresponding to each sigma point is calculated according to the following formula:

$$w_m^0 = \frac{\lambda}{n+\lambda} \tag{18}$$

$$w_c^0 = w_m^0 + (1 - \alpha^2 + \beta) \tag{19}$$

$$w_m^i = w_c^i = \frac{1}{2(n+\lambda)}, i = 1, \dots, 2n \tag{20}$$

$$\lambda = \alpha^2(n+\kappa) - n \tag{21}$$

$$\kappa \approx 3 - n \tag{22}$$

which: w_c^0 is the weight of the first row in the covariance matrix.
w_m^0 is the weight of the first row in the formula (23).

w^i is the weight of the remaining rows.

$\kappa \geq 0$; $\alpha \in (0,1]$;$\beta = 2$ is the optimal choice for the Gaussian distribution

The State-space model.

The Kalman filter is a powerful model-based quantitative technique for identifying structural defects. Applying mathematics to the Unscented Kalman filter, The transition matrix of a continuous system in the general form of the state-space model can be defined as follows:

$$\dot{x}(t) = f(x(t), u(t), w(t)) \tag{23}$$

$$Y(t) = h\big(x(t), u(t), v(t)\big) \tag{24}$$

In which : $\dot{x}(t)$ is a state-space vector; f and h are the conversion function and the measure function; $u(t)$ and $v(t)$ are the process noise vector and measurement noise vector, respectively. To estimate the dynamic properties of the system, such as stiffness and damping, UKF will be used to process the response data obtained from the system. Considering the discrete state-spaces model functions, we have:

$$x_{k+1,k} = F(x_k, u_k, w_k) \tag{25}$$

$$Y_k = H\big(x_{k+1,k}, u_k, v_k\big) \tag{26}$$

In which: $F(x_k, u_k) = x_k + \int_{k\Delta t}^{(k+1)\Delta t} f\big(x(t), u(t)\big)dt$ and $H = h$, Δt is the time step. x_k is a state variable vector and w_k is a discrete process of white noise, Y_k is the measurement vector, v_k is a discrete measurement process noise vector, and R is the covariance value.

UKF is similar to the traditional linear Kalman filter consisting of two steps, measurement and time-step updating. It would allow calculating the Kalman gain in the measurement step, updating the state vector with the estimated error, and using it for the next iteration. The above integral can be solved numerically by using RK4th.

Measurement Steps:

The measurement steps include calculating the state vector and the output vectors covariance matrix and calculating the Kalman gain as follows:

- Calculating Sigma points through the propagation function F:

$$\chi_{k+1|k} = F(\chi_k, u_k) \tag{26}$$

- The estimate state vector:

$$\hat{X}_{k+1}^{-} = \sum_{i=0}^{2n} w_m^i \, \chi_{k+1|k}^i \tag{27}$$

- Estimate the state error covariance matrix:

$$P_{k+1}^{XX} = \sum_{i=0}^{2n} w_c^i \big[\chi_{k+1|k}^i - \hat{X}_{k+1}^{-} \big]\big[\chi_{k+1|k}^i - \hat{X}_{k+1}^{-}\big]^T + Q_k \tag{28}$$

- Calculate the matrix of Sigma points of the measurement variable:

$$\Upsilon_{k+1|k} = \mathrm{H}\big(\chi_{k+1|k}, v_k\big) \tag{29}$$

- Estimate the measurement variable:

$$Y_{k+1}^{-} = \sum_{i=0}^{2n} w_m^i \, \Upsilon_{k+1|k}^i \tag{30}$$

- Estimate the error covariance matrix of the measurement variable:

$$P_{k+1}^{YY} = \sum_{i=0}^{2n} w_c^i \big[\Upsilon_{k+1|k}^i - Y_{k+1}^{-} \big]\big[\Upsilon_{k+1|k}^i - Y_{k+1}^{-}\big]^T + R_k \tag{31}$$

- Estimate the error covariance matrix between the state estimate and the measurement variable

$$P_{k+1}^{XY} = \sum_{i=0}^{2n} w_c^i \left[\chi_{k+1|k}^i - \hat{X}_{k+1}^- \right] \left[\Upsilon_{k+1|k}^i - Y_{k+1}^- \right]^T \tag{32}$$

Calculation Kalman gain:

$$K_{k+1} = P_{k+1}^{XY} \times (P_{k+1}^{YY})^{-1} \tag{33}$$

In processing the equation of state, there will always be disturbances in the data, calculations or estimation errors or uncertainties that lead to incorrect evaluation of the output. This disturbance can come from objective or subjective causes.

The time update step:

$$\hat{x}_{k+1} = \hat{X}_{k+1}^- + K_{k+1}(y_{k+1} - Y_{k+1}^-) \tag{33}$$

Estimate the error covariance matrix for the next iteration:

$$P_{k+1} = P_{k+1}^{XX} - K_{k+1} P_{k+1}^{YY} K_{k+1}^T \tag{34}$$

With time steps k is increasing and $\hat{x}_{k+1}$ is the estimated state vector of the dynamic system. Then, P_{k+1} and $\hat{x}_{k+1}$ will be used for the next iteration to calculate the new Sigma points. Q_k the state error covariance matrix (estimate error), R_k is the measurement covariance matrix (measurement error) and K_{k+1} is the Kalman gain.

In which, K_{k+1} considered as weighted based on a balanced comparison between estimation and measurement errors.

3 Results and discussion

3.1 Simulation results of BFDS vibration responses by RK4th.

The system dynamics model of BFDS is proposed as shown in Fig.1 along with Eq.(1) expressed in 4-dofs form as follows:

$$[M]\begin{Bmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \\ \ddot{x}_3(t) \\ \ddot{x}_4(t) \end{Bmatrix} + [C]\begin{Bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{Bmatrix} + [K]\begin{Bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{36}$$

In which the matrices [M], [K], [C] are defined in Table. 1. To apply the RK4th method to investigate the responses to the system, we first need to choose a suitable sampling frequency f_s. Based on some previous experience, we choose $f_s = 5 \div 20\; max(f_n)$, in which f_n is the natural frequency of the system. Apply the eigenvalue problem in terms of Eq.(3), the frequency in the first four vibration modes: $\{f_1 = 104.34\, Hz;\; f_2 = 266.54\, Hz;\; f_3 = 1082.4\, Hz;\; f_4 = 1443.8\, Hz\}$. From there, it is possible to choose the sampling frequency $f_s = 10000Hz$. With the data input are dynamic parameters and an instantaneous force of 20N on the first generalized coordinate, the response of the mechanical system is simulated by RK4th during the survey time $t = 2s$. The responses of the machanical system are shown in Fig. 2 - 4:

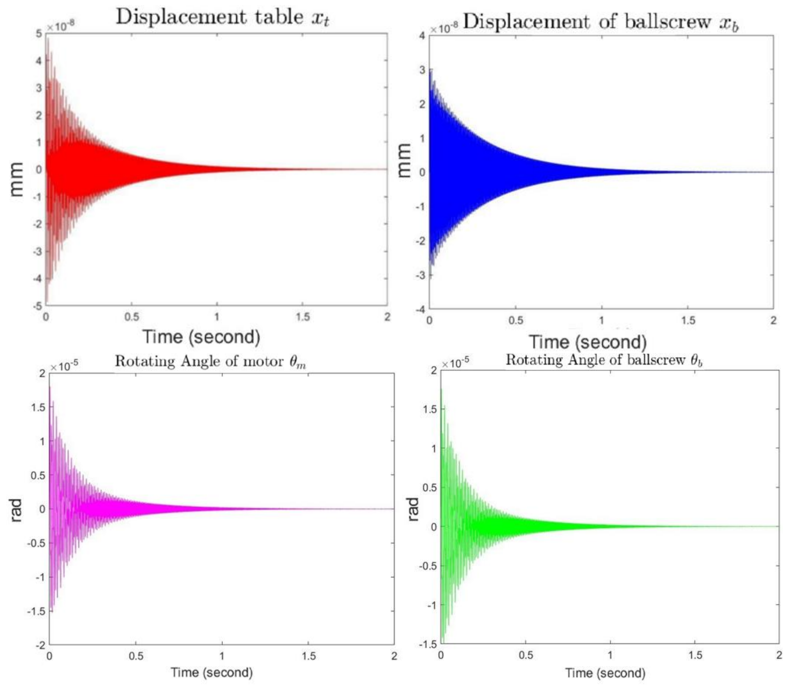

Fig. 2. History of generalized cdisplacement by RK4th

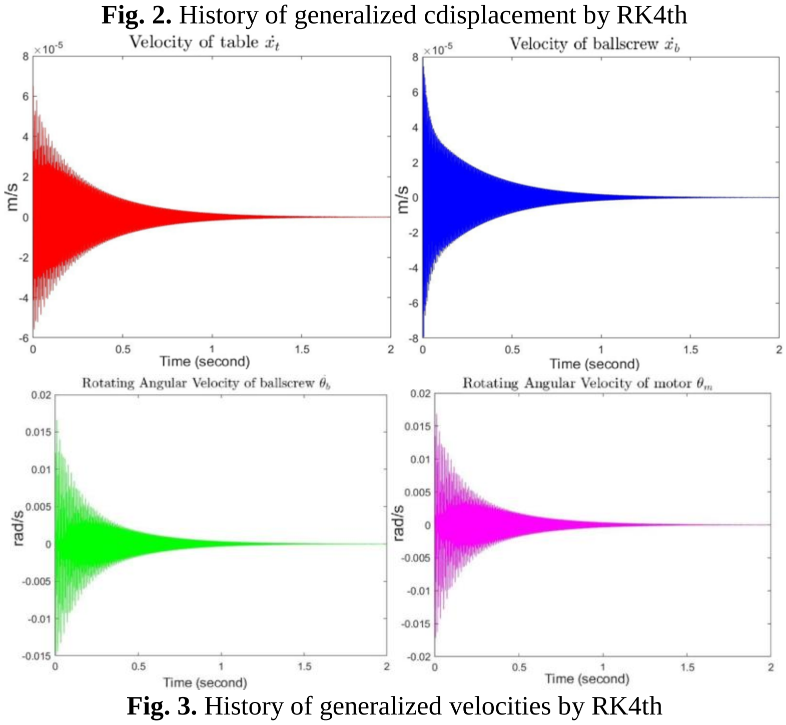

Fig. 3. History of generalized velocities by RK4th

- Acceleration response:

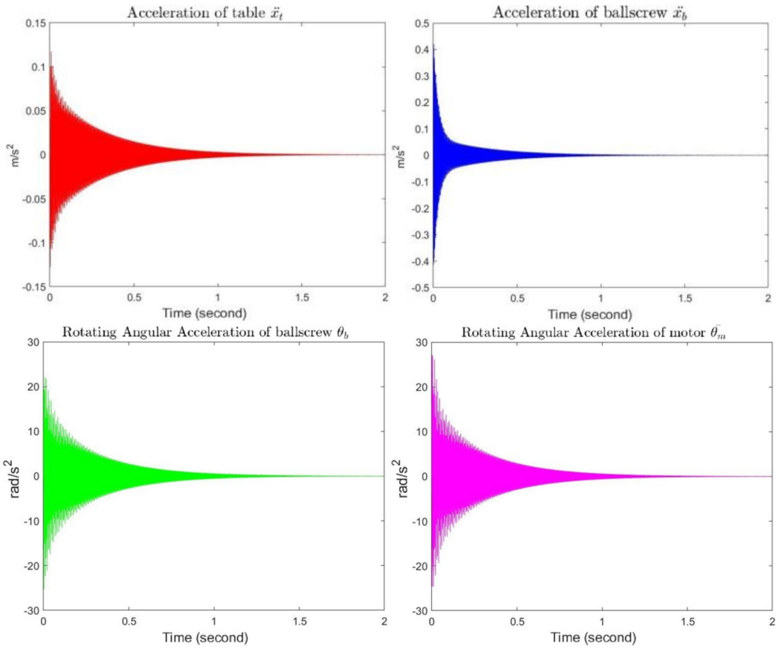

Fig. 4. History of generalized accelerations by RK4th

3.2 The UKF results

Based on the dynamic model, the state-space equation of the 4-dofs BFDS system applied to UKF is shown as follows:

$$\begin{aligned} X &= \{x_t \quad x_b \quad \theta_b \quad \theta_m \quad \dot{x}_t \quad \dot{x}_b \quad \dot{\theta}_b \quad \dot{\theta}_m \quad k_1 \quad k_2 \quad k_3 \quad k_4 \quad B_t \quad B_b \quad Q_b \quad Q_m\}^T \\ &= \{x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_7 \quad x_8 \quad x_9 \quad x_{10} \quad x_{11} \quad x_{12} \quad x_{13} \quad x_{14} \quad x_{15} \quad x_{16}\}^T \end{aligned} \tag{37}$$

$$\dot{X} = \{\dot{x}_t \quad \dot{x}_b \quad \dot{\theta}_b \quad \dot{\theta}_m \quad \ddot{x}_t \quad \ddot{x}_b \quad \ddot{\theta}_b \quad \ddot{\theta}_m \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\}^T \tag{38}$$

Measurement equation $Y(t) = h\big(X(t), v(t)\big)$ is selected according to the responses that we want the estimate to be compared with the input response data. Perform discretization with $t = k\Delta$. To represent the state-space equation of the next k step, the state transition function $F(x_k, u_k) = x_k + \int_{k\Delta t}^{(k+1)\Delta t} f\big(x(t), u(t)\big)dt$ as follows:

$$x_{k+1|k} = F(x_k, u_k) = x_k + \int_{k\Delta t}^{(k+1)\Delta t} f(x(t), u(t)) = \begin{Bmatrix} x_1 + x_5\Delta t \\ x_2 + x_6\Delta t \\ x_3 + x_7\Delta t \\ x_4 + x_8\Delta t \\ x_5 - \Delta t/M_t\,(x_{12}x_5 + x_9x_1 - x_9x_2 - x_9\beta x_3) \\ x_6 - \Delta t/M_b\,(x_{13}x_6 - x_9x_1 + (x_9 + x_{10})x_2 - x_9\beta x_3) \\ x_7 - \Delta t/J_b\,(x_{14}x_7 - x_9\beta x_1 + x_9\beta x_2 + (d_r\alpha\beta x_9 + x_{11})x_3 - x_{11}x_4) \\ x_8 - \Delta t/J_m\,(x_{15}x_8 - x_{11}x_3 + x_{11}x_4) \\ x_9 + 0 \\ x_{10} + 0 \\ x_{11} + 0 \\ x_{12} + 0 \\ x_{13} + 0 \\ x_{14} + 0 \\ x_{15} + 0 \\ x_{16} + 0 \end{Bmatrix} \tag{39}$$

The measurement equation has been discretized $Y_k = H(X_{k+1|k}, v_k)$ where H is a measure similar to h. Thus, based on the state-space equations X_k, the measurement equation Y_k and the state transition function F, the UKF process as shown in Eq.(26)-(34).

3.2.1 State estimate.

The input data is based on Table.1 include [M], [K], [C] and the excitation force as known in the process of simulating the vibration response by RK4th. Next, we choose the initial condition of UKF as follows:

$$\alpha = 1/2, \beta = 2, \kappa = 3, \lambda = -11.25 \tag{40}$$

$$w_n^0 = -2.368, w_c^0 = 0.38, w_m^i = w_c^i = \frac{2}{19} \tag{41}$$

$$\Delta t = \frac{1}{f_s} = \frac{1}{10000} = 0.0001s \tag{42}$$

$$X_0 = \{\overline{x_1} \quad \overline{x_2} \quad \overline{x_3} \quad \overline{x_4} \quad \overline{\dot{x}_1} \quad \overline{\dot{x}_2} \quad \overline{\dot{x}_3} \quad \overline{\dot{x}_4} \quad k_1 \quad k_2 \quad k_3 \quad k_4 \quad c_1 \quad c_2 \quad c_3 \quad c_4\}^T \tag{43}$$

$$P_0 = \mathrm{diag}\begin{bmatrix} \mathrm{cov}(x_1), \mathrm{cov}(x_2), \mathrm{cov}(x_3), \mathrm{cov}(x_4), \mathrm{cov}(\dot{x}_1), \mathrm{cov}(\dot{x}_2), \mathrm{cov}(\dot{x}_3), \\ \mathrm{cov}(\dot{x}_4), 0,0,0,0,0,0,0,0 \end{bmatrix} \tag{44}$$

The measurement variables applied to the mechanical system include displacement, velocity and acceleration responses calculated based on the RK4th method. Process noise matrix: $Q = 10^{-4} \times \mathbf{I}_{16\times16}$. Measurement noise matrix: $R = 10^{-9} \times \mathbf{I}_{12\times12}$. The results of the response estimation of the system are shown in Fig. 5 6 7.

- Estimation of displacement response by UKF

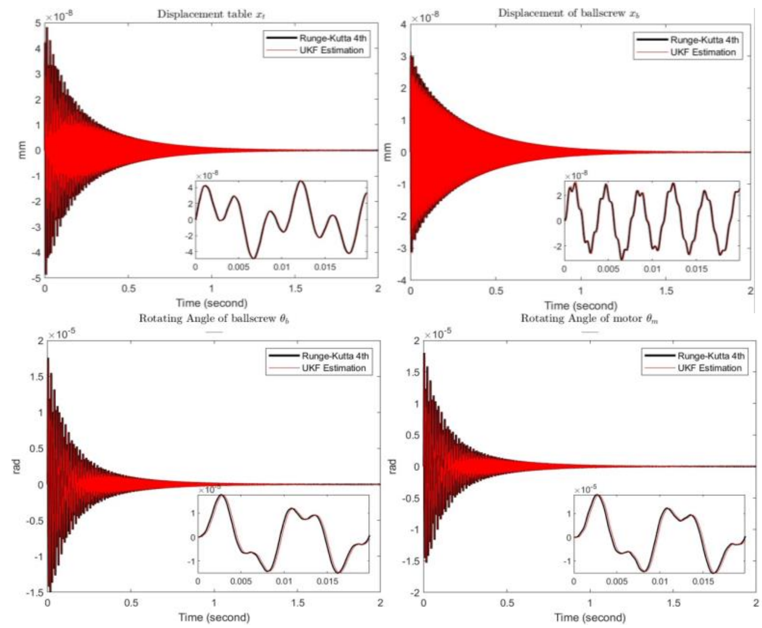

Fig. 5. The estimation of generalized displacement by UKF

- Estimation of velocity response by UKF

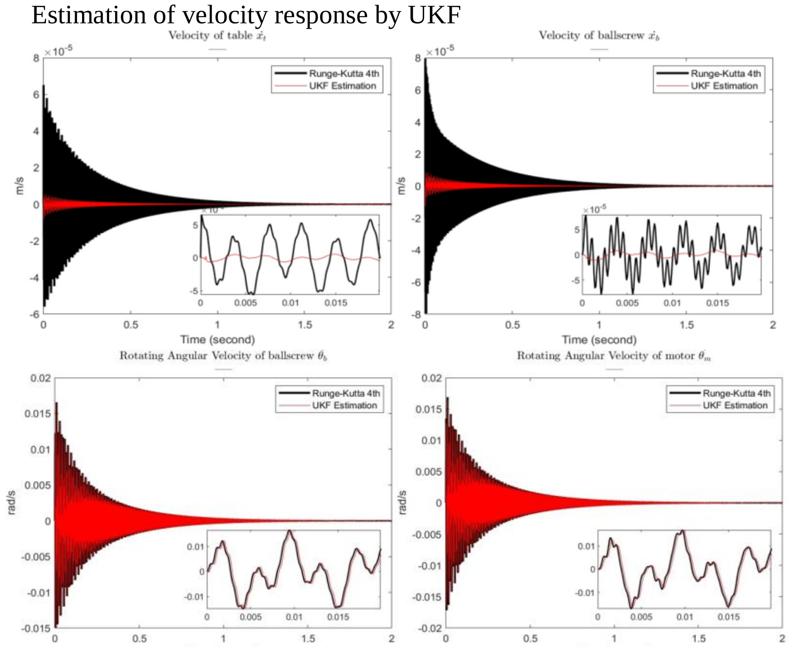

Fig. 6. The estimation of generalized velocities by UKF.

- Estimation of acceleration response by UKF.

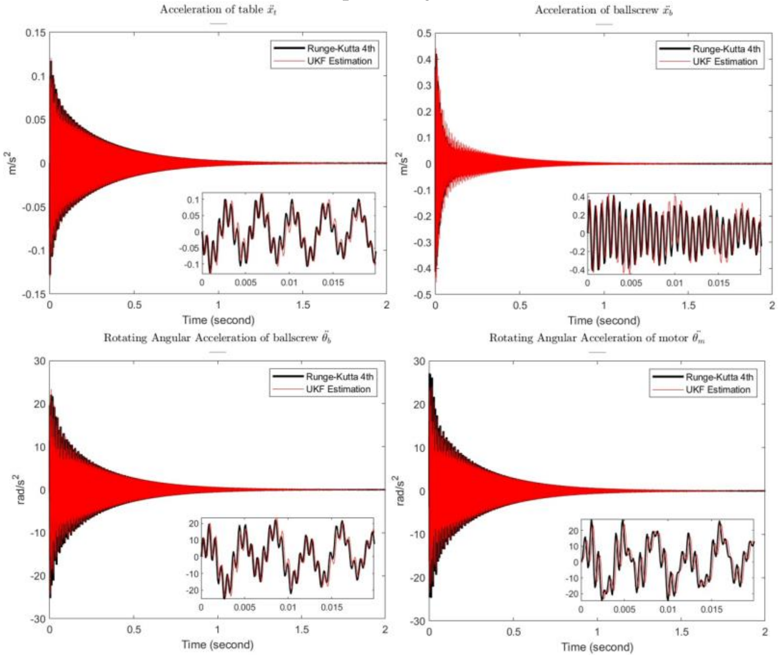

Fig. 7. The estimation of generalized accelerations by UKF

3.2.2 Dynamic parameter estimation.

The process of estimating dynamic parameters [K] and [C] depends a lot on the selection of initial data, in which three factors have the most significant impact on the performance and accuracy of UKF, including:

- An initial error covariance matrix P_0.
- Process noise matrix Q.
- Measurement noise matrix R.

To investigate the influence of these three factors in the estimation process, we provide the parameter values of stiffness, damping and measurement variables such as displacement response, velocity, and acceleration. Then operate UKF and comment on the results regardless of whether they converge. So for each case, when we change different parameters of the mechanical system (stiffness or damping or both), there will exist parameter sets containing corresponding P_0, Q and R objects that we need to investigate. They are represented as follows:

$$P_0 = \begin{bmatrix} cov(x1) & 0 & & & \cdots & & & 0 \\ & cov(x2) & & & & & & \\ & & \ddots & & & & & \\ & & & \ddots & & & & \\ & & & cov(v3) & & & & \vdots \\ \vdots & & & & \left(\sigma_{k_1}^{error}\right)^2 & & & \\ & & & & & \left(\sigma_{k_2}^{error}\right)^2 & & \\ & & & & & & \ddots & 0 \\ 0 & & \cdots & & & & 0 & \left(\sigma_{c_4}^{error}\right)^2 \end{bmatrix}_{12\times 12} \tag{45}$$

$$Q = diag\left(\left[u_p\right]\right)_{16\times 16} \tag{46}$$

$$R = diag([u_m])_{12\times 12} \tag{47}$$

The initial conditions are set up the same as the steps of State Estimation. The parameter estimation results by UKF are shown as follows:

- The estimation of the stiffness parameter is shown in Fig. 8.

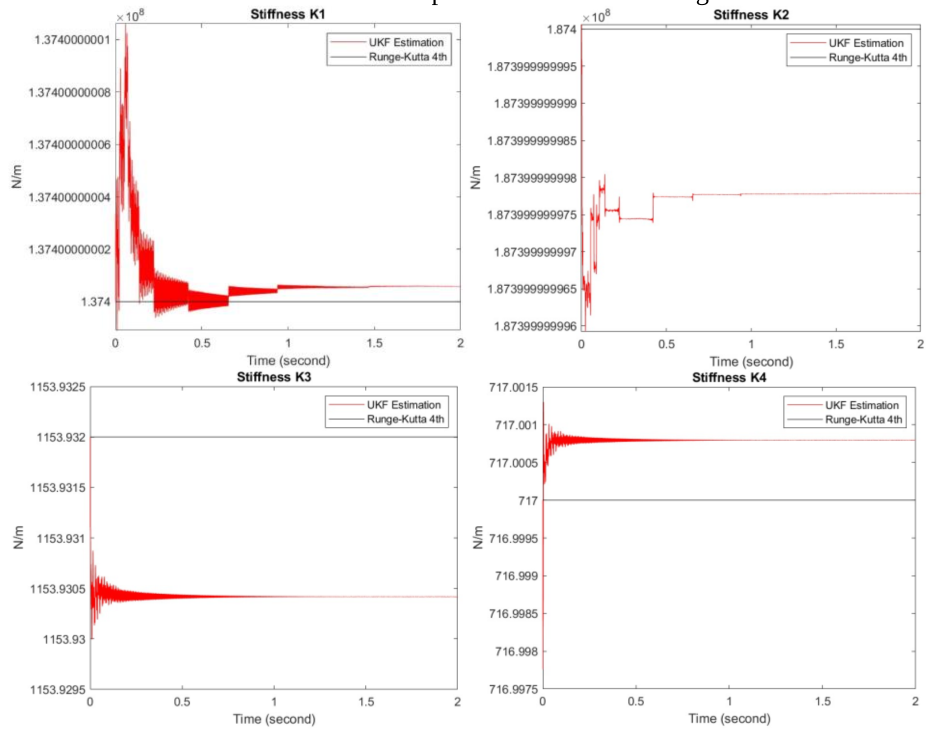

Fig. 8. The estimation of the stiffness parameter by using UKF

- The estimation of the damper parameter is shown in Fig. 9.

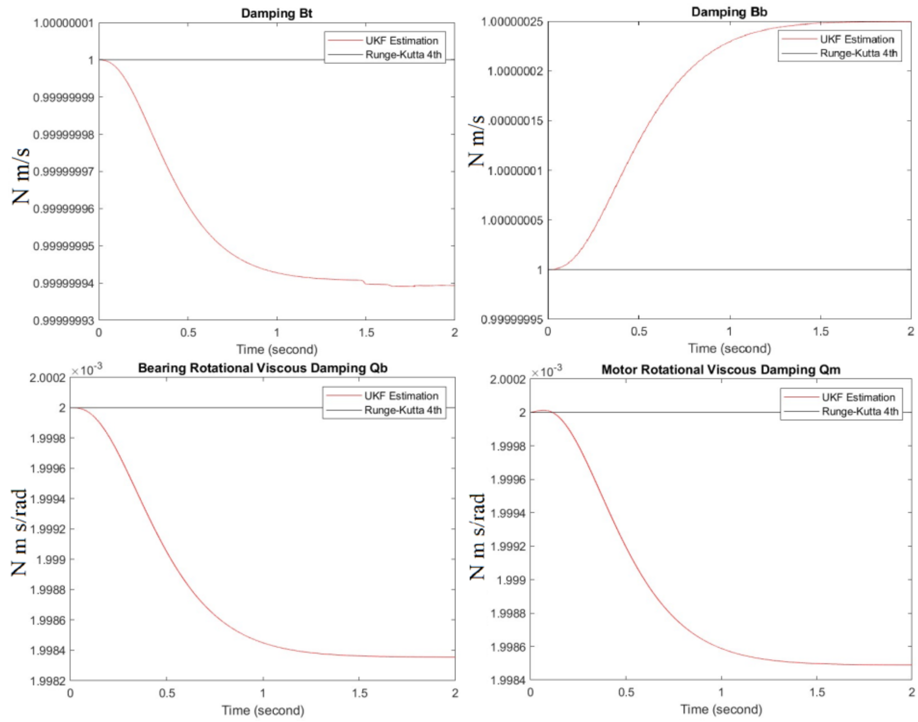

Fig. 9. The estimation of the damper parameter by using UKF.

3.3 Compare and evaluate the estimated results.

3.3.1 Response estimation results by using UKF.

Based on the diagrams, Fig. 2 – Fig. 7 shows the results of simulated responses by the RK4th method and response estimation results of UKF; the similarities can be seen through visual observation.

The correlation coefficients are calculated to evaluate the correlation between the results of RK4th simulated responses and the UKF estimated. The correlation coefficients are calculated and listed in Tables. 2.

Table. 2. Correlation coefficients between RK4th and UKF at the generalized responses.

Response estimation	Generalized Coordinates			
	x_t	x_b	θ_b	θ_m
The correlation coefficients (r)	0.988	0.982	0.979	0.977
Response estimation	Generalized Velocity			
	$\dot{x}_t$	$\dot{x}_b$	$\dot{\theta}_b$	$\dot{\theta}_m$
The correlation coefficients (r)	0.673	0.741	0.957	0.956
Response estimation	Generalized Acceleration			
	$\ddot{x}_t$	$\ddot{x}_b$	$\ddot{\theta}_b$	$\ddot{\theta}_m$
The correlation coefficients (r)	0.933	0.889	0.929	0.897

Based on correlation coefficients (r) according to Table. 2, it can be remarked that the response estimation results have a similarity. The correlation coefficients (r) between responses are quite high, with the average value $\bar{r} = 0.908$. This non-uniformity warns that there are some problems related to the parameter set containing the objects P_0, Q and R.

Furthermore, Fig. 5 6 7 shows deviations at the initial loop times and rapid convergence based on the short duration (averaged from 1÷1.5s=) and slope of the graphs.

The comments from the above response estimation results have proved that the UKF model can be considered to apply to complex systems such as BFDS.

3.3.2 Estimation of dynamic parameters by UKF

The UKF parameter estimation method has shown superiority over other numerical methods. The error percent is calculated and listed in Table. 3 based on the dynamic parameter estimation (Fig. 8-9).

Table. 3. The parameter estimated error percent.

Parameter estimation	K_1	K_2	K_3	K_4	B_t	B_b	Q_b	Q_m
Percent (%)	$5.23e^{-10}$	$1.27e^{-9}$	$1.36e^{-4}$	$1.09e^{-4}$	$4.66e^{-6}$	$1.82e^{-5}$	$6.17e^{-2}$	0.055

It was found that there was no significant difference between the stiffness $[K]$ and damping $[C]$ estimates compared with the input values. The percentage of deviations between the final UKF estimated values and the data input values clearly shows this.

In terms of convergence, the stiffness parameter estimates show faster convergence (within $0.5 \div 1s$) than damping (within $1.5 \div 2s$). This can be explained through the parameter set containing the objects P_0, Q and R suitable for stiffness estimation but not suitable for damping.

The main reason for the effectiveness of UKF is that it is a powerful filter to estimate dynamic parameters while having only a finite number of state responses input data. Therefore, UKF results will estimate better when there are many input responses.

4 Conclusion

This paper has presented the survey method and system identification for BFDS. BFDS is modelled as a mechanical system with 4-DoFs, and dynamic parameters are determined that are directly related to the vibration responses of the system. This paper has investigated and estimated that the vibration response of the system, such as displacement, velocity, and acceleration, could be determined using two numerical methods, including the RK4th and UKF methods. In addition, UKF has demonstrated the ability to estimate the dynamic parameters of the system accurately, which is also considered a novelty of the research topic. The obtained results and some comments are as follows:

- System identification through the vibration responses of a complex system, including rotational and translational motion such as BFDS, is one of the topics of great interest. Several methods of modal analysis through system dynamics modelling have achieved remarkable success. Even so, these methods have some limitations, depending on the degrees of freedom, how the mechanical system is modelled, and how the dynamic parameters are calculated. It makes it difficult to determine the system by experiment. Consequently, numerical methods such as RK4th and UKF have been applied to the dynamic model, allowing us to estimate the state of the BFDS based on vibrational responses corresponding to each generalized coordinate. The high correlation between the two numerical methods shows the correctness of this paper.

- This study has achieved initial success in applying UKF by accurately estimating the dynamic parameters of the system like [K] and [C]. However, when using UKF in the experiment, it will be necessary to consider specific cases. Certain limitations exist in collecting experimental measurement responses as input data to UKF, so the amount of input response data will be limited. In the experiment, it is easy to measure the acceleration responses through the accelerometer sensor system, which is placed in suitable positions on the BFDS, but the rotational acceleration responses will sometimes be difficult.

- Besides, for the UKF model to estimate accurately, some conclusions are drawn as follows: the first is the selection of the number of degrees of freedom for the mechanical model and the dynamic parameter; consider the processes of establishing variables in the state-space equation; consider the measurement or response variables and excitation forces used as input for UKF; the set of weight parameters for the covariance matrices, the process noise matrix, and the measurement noise matrix need to be carefully considered and set up.

- The scope of this paper's research is limited to surveying through numerical simulation to provide a new approach for applying UKF to the BFDS system. Some advantages and disadvantages are also mentioned in the above comments. From there, it opens up the scope of future research, aiming to establish an efficient and suitable data collection measurement system to ensure the quality of the input dataset for UKF. Besides, it is necessary to overcome the limitation on the number of input measurement data for the UKF to estimate dynamic parameters more easily and approximate the experiment.

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