Analysis of free cable vibration with material damping for an applied cable-stayed bridge

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Abstract. Recently, monitoring cable tension has been an important problem in assessing the condition of cable-stayed structures. Many factors that affect cable tension, such as flexural rigidity, sag, lay angle, and complicated boundary conditions, were studied. However, the influence of damping has rarely been considered. In this paper, the mathematical model of the damped vibration of the cable has been studied to find the effect of material damping on tension. The experimental results from vibration data of cables in the cable-stay bridge are used to verify the relationship between frequency, damping, and tension. This study aims to use vibration techniques to increase accuracy in tension determination and condition assessment of cables.

Keywords: Damping properties, Cable tension, Natural frequency, Cable-stayed bridge, Cable vibration.

1 Introduction

Owning large rivers and the rapid traffic development makes bridge structure status research in Vietnam necessary. The cable-stayed bridges are one of the solutions for across large rivers; they include many parts such as one or more pillars, bridge deck, beams, and the stayed cable. The cable is a tensile part from an excited source, so the material properties and geometry directly influence these special mechanical structures. One of the influent factors on vibration is internal damping from the mechanical properties of a material. This damping mechanism helps structural cable-stayed bridges obtain stability under a working load. An investigation for damping of the beam by viscoelastic material was carried out in Jayant Saran Lai's thesis in 1966; this investigation described damping mechanisms with discretized models and damping parameters are constants [1]. Since 1989, D. J. Inman et al. have continuously studied the damping mechanisms on beam models, whereas many damping models have been given. However, these models are only suitable for laboratory studies [2-3]. In 2000, Der-Wen Chan ae al. proposed a mathematical procedure on damping in a time domain to analyse the structural response and had comparisons with the other methods [4]. It was not until 2018 that Wen-Hwa Wu et al combinedly studied the damping and tension to monitor tensile structures, especially on cables by extending the taut string theory to determine the cable tension and did both experimental and practical models at some suspension and stayed bridges [5-6]. In 2019, Xiang Shi et al researched inertial dampers of cable dynamic to use shapes of mode for optimally designed dampers to dissipate the energy of cable [7]. Yan et al proposed to consider mode shapes to estimate tension force of cable dealing with complicated boundary conditions in 2019 [8]. Also this year, research about controlling the oriented mode for the response of cable with damping was performed by Javanbakht et al [9]. Aiko Furukawa et al. suggested that the proposed novelty method to determine the tension, bending stiffness, and damper of the simultaneous cables from the natural frequency [10]. However, these methods have difficulty applying practical problems and have been costly to perform with a high exact value. In practical terms, damping is usually small, and its exact influences are simplified and ignored, so the researchers do little research on how internal damping influences the cables in the traffic field. In this study, a mathematical model for free vibration with material damping has been studied, calculated, and applied with property material to the tensile cable system for the Phu My Bridge, which is one of the cable-stayed bridges in Ho Chi Minh City. This study is expected to provide a new applied mathematical model to determine the cable tension of the cable-stayed bridge.

2 The mathematical model

The cable has been modeled as an extended viscoelastic beam with length L, and dynamic excited load of $q(x,t)$ per unit length as shown in Fig. 1. Whereas the governing differential equation of motion of cables as beam model with viscoelastic damping, derived as [11]:

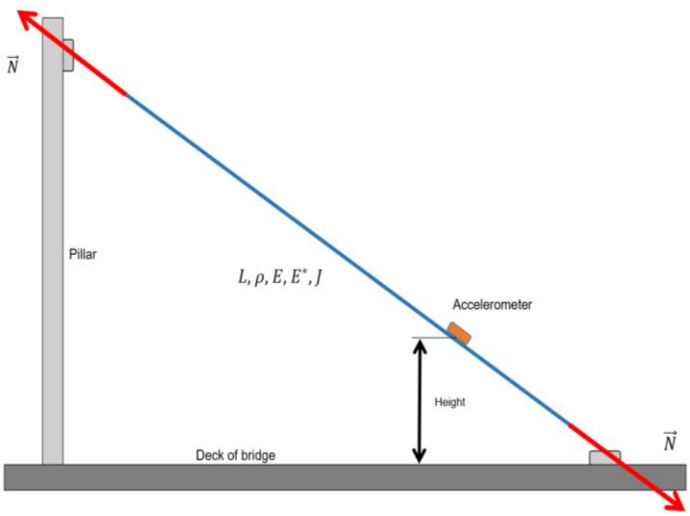

Fig. 1. The cable modelling as the beam.

$$\frac{\partial^2}{\partial x^2}\left[EJ(x)\frac{\partial^2 u(x,t)}{\partial x^2}\right]+\rho(x)\frac{\partial^2 u(x,t)}{\partial t^2}-N(x)\frac{\partial^2 u(x,t)}{\partial x^2}+E^*J(x)\frac{\partial^5 u(x,t)}{\partial x^4 \partial t}=q(x,t) \qquad (1)$$

Where: E is Young's modulus of material (Pa), $J(x)$ is the variable inertia of the beam (m^4), $\rho(x)$ is the weight density per unit of length *(kg/m)*, $N(x)$ is axial force (*N*), E^* is the viscoelastic damping factor (*Ns/m*^{2}), and $u(x,t)$ is the equation of transverse motion.

To simplify the governing differential equation for the free vibration of the cable ($q(x,t)$= 0), properties of material as E, J, ρ, N, E^* are simply assumed as constants per unit of length. Then, Eq. 1 is given by:

$$EJ\frac{\partial^4 u(x,t)}{\partial x^4}+\rho\frac{\partial^2 u(x,t)}{\partial t^2}-N\frac{\partial^2 u(x,t)}{\partial x^2}+E^*J\frac{\partial^5 u(x,t)}{\partial x^4 \partial t}=0 \qquad (2)$$

The classified separation of variables method is applied to divide the transverse deflection function $u(x,t)=W(x)T(t)$ into mode shape $W(x)=Ce^{sx}$ and response function $T(t)=De^{a\omega t}$. The technique is determined as follows:

$$\frac{EIW(x)^{(4)}}{-\rho W(x)+\frac{NW(x)''}{\omega^2 a^2}}=\frac{\ddot{T}(t)}{T(t)+\frac{E^*}{E}\dot{T}(t)}=\lambda \qquad (3).$$

Where: *C*, D is the constant of equations, $a=-\zeta+i\sqrt{1-\zeta^2}$ with equivalent damping ratio ζ(i is complex number), and $\lambda=-\omega^2$ is the eigenvalue of the differential equation [12].

From Eq. (3), there are:

$$EIW''''(x)+\frac{N}{a^2}W''(x)-\omega^2\rho W(x)=0 \qquad (4)$$

$$\ddot{T}(t)+\frac{E^*\omega^2}{E}\dot{T}(t)+\omega^2 T(t)=0 \qquad (5)$$

In recent assumptions, the cable model models a uniform hinged-hinged beam at both ends, which means $x=0$, as well as $x=L$. The solution of the differential equation was performed by [12]:

$$W(x)=C_1e^{s_1x}+C_2e^{-s_1x}+C_3e^{s_2x}+C_4e^{-s_2x} \qquad (6)$$

Where: $s_1^2 = s_2^2 = \frac{-N\pm\sqrt{4EI\rho\omega^2a^4+N}}{2EIa^2}$

$$T(t)=D_1e^{t\left(\frac{\omega\sqrt{E^*\omega^2-4E}-E^*\omega^2}{2E}\right)}+D_2e^{t\left(\frac{-\omega\sqrt{E^*\omega^2-4E}-E^*\omega\partial^2}{2E}\right)} \qquad (7)$$

With Eq. 7, the critical viscoelastic damping factor E_C^* and the damping ratio ζ are charged with finding by:

$$E^*\omega^2-4E=0 \qquad (8)$$

$$E_C^*=\frac{E}{\pi f_n} \qquad (9)$$

$$\zeta = \frac{\pi f_n E^*}{E} \quad (10)$$

From Eq. 6 and Eq. 7, the damping factor has been determined by:

$$E^* = \frac{E}{\pi}\sqrt{\frac{4\left(\frac{f_n}{n}\right)^2 - 4\left(\frac{f_m}{m}\right)^2 - EI\left(\frac{\pi}{L}\right)^2 (n^2 - m^2)}{8\left(\frac{f_n^4}{n^2} - \frac{f_m^4}{m^2}\right) - 2EI\left(\frac{\pi}{L}\right)^2 (n^2 f_n^2 - m^2 f_m^2)}} \quad (11)$$

Where: n, m are the first n^{th} and m^{th} of modal frequency

According to the boundary conditions hinged-hinged $sl = n\pi$, Eq. 6 has been used for the mathematical model denoted:

$$\sqrt{\frac{-N \pm \sqrt{4EI\rho\omega^2 a^4 + N}}{2EIa^2}} = \frac{n\pi}{L} \quad (12)$$

And then, the tension of the cables is determined as follows:

$$N = \left[\rho\left(\frac{2Lf_n}{n}\right)^2 - EI\left(\frac{n\pi}{L}\right)^2\right]\left[2\left(\frac{\pi f_n E^*}{E}\right)^2 - 1 - 2i\frac{\pi f_n E^*}{E}\sqrt{1 - \left(\frac{\pi f_n E^*}{2E}\right)^2}\right] \quad (13)$$

Generally, the natural frequency depends on material properties, geometry, and damping factors. When the cable-stayed bridge is under service load, the cable tension has a value as the modulus of Eq. 13. In the next section, an actual model measured vibration to evaluate the cable tension with viscoelastic damping.

3 Analysis of the proposed method

A real experiment was performed at some cables of the Phu My bridge, as shown in Fig. 2, Ho Chi Minh City, in 2017. The primary practical devices include accelerometers, a laptop to connect to the accelerometers, and a ladder,...

Fig. 2. Vibration measurement on the cables of The Phu My stayed bridge.

The cable material parameters have been collected from a design document for four cables in the Phu My bridge before the experimental measurement, as shown in Table 1. In the experiment, accelerometers were installed for each cable, with the height from the bridge desk was about 2.75 meters. Due to mathematical transformation, the modal frequencies have been determined from vibration spectrum (Figure 3) and synthetized in Table 2. And then, using the formulas from the previous section, the analysis result, which includes the viscoelastic damping parameter and the cable tension, is given in Table 3 and Table 4.

Table 1. Material parameters of cables of the Phu My Stayed Bridge.

Cable No	Density mass Kg/m	Length m	Inertia m^4	Young Modulus N/m^2
C2102	31.86	68.13	8.36×10^{-6}	1.97×10^{11}
C2207	41.30	101.22	1.37×10^{-5}	
C2212	53.10	148.12	2.18×10^{-5}	
C2215	60.18	179.22	2.47×10^{-5}	

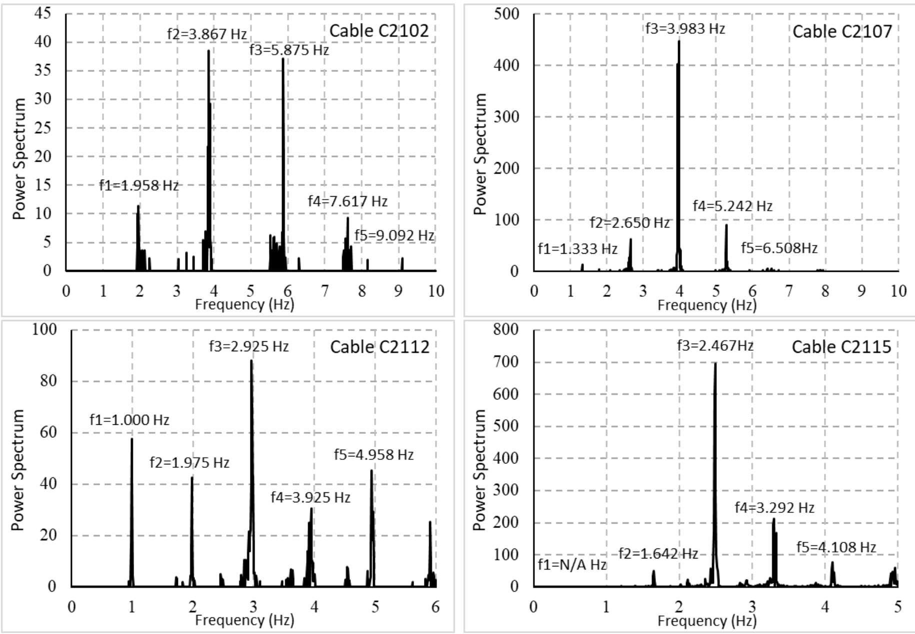

Fig. 3. The vibration spectrum of some cable in Phu My Bridge

Table 2 collects data from three measured times within 60 seconds per repeat for each cable.

Table 2. The modal frequency.

Cable No	C2102			C2207			C2212			C2215		
Time	1	2	3	1	2	3	1	2	3	1	2	3
Mode 1	1.958	1.933	1.950	1.333	1.208	1.325	1.000	0.992	0.992	N/A	N/A	N/A
Mode 2	3.867	3.850	3.900	2.650	2.667	2.650	1.975	1.983	1.975	1.642	1.650	1.608
Mode 3	5.875	5.875	6.050	3.983	3.983	3.983	2.975	2.967	2.983	2.467	2.475	2.408
Mode 4	7.617	7.600	7.717	5.242	5.292	5.283	3.925	3.950	3.933	3.292	3.292	3.200
Mode 5	9.092	9.533	9.308	6.508	6.550	6.583	4.958	4.942	4.958	4.108	4.125	3.958

From the data in Table 1 and 2, they are using Eq. 12 and Eq. 13 to evaluate average damping factors and ratios. The results are given in Table 3 below.

Table 3. The viscoelastic damping of Phu My Bridge

Cable No		Damping factor $E^* \ (Ns/m^2)$	Damping ratio ζ
C2102	1	5.677×10^9	0.4748
	2	5.585×10^9	0.4687
	3	5.565×10^9	0.4731
C2207	1	8.160×10^9	0.4692
	2	8.102×10^9	0.4643
	3	8.160×10^9	0.4692
C2212	1	9.090×10^9	0.4672
	2	10.849×10^9	0.4671
	3	10.857×10^9	0.4672
C2215	1	$12{,}18 \times 10^9$	0.5286
	2	12.91×10^9	0.5287
	3	13.33×10^9	0.5306

According to Table 3, the higher the number of strands of the cable, the greater the material damping factor. However, the damping ratio is less dependent on the number of strands. This demonstrates that the loss of vibrational energy due to the friction between the strands is related to the material damping properties in the beam model. However, more studies are needed to confirm this property.

With the viscoelastic damping from cable material, the tensions have been evaluated by Eq. 15 and are clearly shown in Table 4. As a result of this procedure, the solutions of calculations consist of real and imaginary numbers. In Table 4, the modulus of the complex number provides direct information about the tension of the cable-stayed bridge.

Table 4. The results of cable tension of Phu My Bridge

Cable No		*This study* (kN)	*Taut string Theory* (kN)	*Error*
C2102	1	2131	2170	1.80%
	2	2153	2192	1.78%
	3	2193	2231	1.70%
C2207	1	2919	2948	0.98%
	2	2837	2866	1.01%
	3	2919	2948	0.98%
C2212	1	4215	4572	7.81%
	2	4207	4564	7.82%
	3	4209	4565	7.80%
C2215	1	4947	5223	5.28%
	2	4979	5256	5.27%
	3	4686	4944	5.22%

Accordingly, the tension of cable from the proposed model are smaller than taut string model because the damping mechanisms have effectively reduced vibration. The analysis results from the cable vibration of the Phu My Bridge showed that The existence of a material damping in the vibration of the cable will be useful for the resistance of the cable under the effect of the traffic load on the bridge. This makes the problem of cable condition monitoring during operation possible. It is necessary to carry out the practical experiment in the future to investigate more about the trend change of both the cable tensions and the material damping.

4 Conclusion

Euler-Bernoulli beam theory with geometry and material properties such as viscoelastic damping, flexural rigidity, and mass is extended and applied to the cable model in this study. The vibration of the cable-stayed bridge has been performed in the fifth-order differential equation with geometry and material properties such as viscoelastic damping, flexural rigidity, and mass. After that, the mathematical technique has been used to separation of variables and solve calculus equations with boundary conditions. Eventually, the cable tension and viscoelastic damping factor were indirectly assessed. The proposed method uses many influential factors on vibration for cable vibration; however, it is inexpensive and convenient to practice in the structural site. This study has suggested that material damping should be considered in the vibration problem of the cable to increase the accuracy in determining the cables' tension.

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