Applying Mixed FEM in Non-linear Dynamic Analysis of Plane Truss with Length Imperfection

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**Abstract.** The article presents the dynamic analysis of plane trusses with the length imperfection of elements. The formulation of the finite element method is established based on a mixed model. To study plane trusses under dynamic loading with element length imperfection taking into account the geometrical nonlinearity, the establishment procedure of the calculation algorithm can be performed by the assumption that the imperfection length is a parameter. In order to deal with the problem of elements with imperfection length, the article proposed an approach based on mixed FEM formulation to solve the trusses with imperfection length of elements subjected to dynamic loading. The paper establishes the dynamic equilibrium equation for the proposed mixed finite element of trusses, formulated based on the compatibility equation considering the geometrical nonlinearity. It applied the Newmark integration and Newton Raphson iteration methods to solve the system of nonlinear dynamic equations of the trusses. The established incremental-iterative algorithm based on these methods is used to write a program for dynamic analysis for trusses with imperfection length of elements. The obtained results verify the effectiveness and accuracy of the proposed mixed FEM formulation in the dynamic analysis of trusses with imperfection length.

**Keywords:** Length imperfection, Mixed FEM formulation, Mixed finite element method, Nonlinear analysis of truss, Dynamic analysis, Plane truss

1. Introduction

Dynamic analysis of structural systems such as trusses has become very important and a concern in many types of research and engineering practical designs [1-9], [11]. In recent years, many studies [1-5] have already mentioned the geometrical nonlinear dynamic problem of the trusses under dynamic loads such as harmonic and impulsive loads. The applied mathematical techniques to solve the problem of nonlinear dynamics of the trusses in almost research are based on the finite element method. Due to manufacturing, transportation and assembly, many truss elements have initial geometrical imperfections, which will significantly impact the dynamic behavior of the plane trusses. In the linear finite element analysis, the initial length imperfection can be easily considered by replacing them with the equivalent external load in the nodes. In [6], the solving algorithm for nonlinear dynamic analysis of trusses used mathematical techniques to deal with imperfect elements, such as the penalty function and Lagrange multiplier methods. There is a new approach of formulation based on the mixed finite element, which is proposed to deal with the mathematical difficulty in solving the system with the length imperfection of the component. Using the mixed finite element formula, which is already presented in the previous study of the authors [7], it is found out the mixed finite truss element with length imperfection takes into account geometrical nonlinearity. With the continuous ideas of the previous study [7], the mixed finite truss element with length imperfection is applied to model the plane truss element with length imperfection under dynamic loading. The system of nonlinear dynamic equilibrium equations for mixed finite truss elements is established by adding inertia and damping forces to the static equilibrium equation in [7] based on the D'Alembert principle. The Newmark integration method is combined with the Newton-Raphson iterative approach to establish an incremental, iterative algorithm to solve the dynamic equilibrium equation of the truss system under dynamic loading. The calculation program is written in MATLAB software based on the proposed incremental, iterative algorithm to study the dynamic behaviour of the plane truss system with the length imperfection under dynamic loading. The written calculation program is applied to implement the numerical investigation for the time-dependent response of the plane truss system under dynamic loading. The results of the numerical analysis show the effectiveness of the mixed finite element formula in solving the nonlinear dynamic problem of the truss system with the length imperfection under dynamic loading. The numerical results show that when the amount of the length imperfection approaches zero, the solution converges to the case of length perfection.

1. Problem Formulation
   1. Dynamic equilibrium equation of plane truss element established by mixed model



**Fig. 1.** Mixed truss element with element’s length imperfection considering geometric nonlinearity

The main idea of establishing a dynamic equilibrium equation of a truss element with length imperfection by adding inertia and damping forces to the static equilibrium equations in [7] is based on the D’Alembert principle. In the article, the authors propose the truss element *e* in the global coordinate system XOY which has unknowns of displacements and forces, as shown in Figure 1. This allows taking into account the length imperfection of the mixed matrix of plane truss elements with consideration of large displacements effectively.

In the research, it is used the following parameters

- Initial length of element;  - the distance between *ith* and *jth* node before deformation ; *L* - the distance between *ith* and *jth* node after deformation; - the amount of length imperfection; A - cross-sectional area of truss element, *E* – elastic modulus of material; *N* - axial load of truss element; - resultant external force at the ith cross-section after deformation (i’); - nodal lumped masses;  - external nodal forces; - inertia, damped and external dynamic loads; is the vector of unknowns, including - nodal displacements in the global coordinate system, - unknown force at the *ith* cross-section after deformation, obviously from equilibrium condition;

The equation of motion according to D’Alembert principle:

 (1)

In the matrix form:

 (2)

Based on the static equilibrium equation in [7] and the D’Alembert principle, the dynamic equilibrium equation of plane truss element with length imperfection and large displacement is obtained as follows:

 (3)

Eq. (3) can be re-written as follows:

 (4)

With following:



Applying the Taylor series expansion, the incremental equilibrium equation of (4) is expressed as follows:

 (5)

Where:

- Vector of incremental displacement unknowns

- Vector of incremental acceleration and vector of incremental velocity;

 - Mass and damping matrix;  - Vector of incremental dynamic load;

 - the mixed matrix of truss element considering the initial length imperfection .

* 1. Dynamic equilibrium equation of plane truss system

According to the finite element method, the dynamic equilibrium equation and the incremental equilibrium equation of the truss system are established based on the indexed assembly of all components corresponding to each truss element in the overall coordinate system, getting:

 (6)

 (7)

In which:



* 1. Solving algorithm

The Newmark integration method [10] is combined with the Newton-Raphson iterative approach [1-9] to solve the system of nonlinear dynamic equations of the trusses (6), (7). The incremental-iterative algorithm established based on a combination of these methods is applied to write a program to calculate the vibrations of truss systems with length imperfection of elements.

According to Newmark [10], the increments of acceleration and velocity are expressed as follows:

 (8)

Replace from (8) into (7) and transform, getting:

 (9)

The incremental equilibrium equation (9) is written in the following form:

 (10)

The Newton-Raphson iterative method is applied to find the solution of equation (10) at each step of time like mentioned in [1,5,6,8].

* 1. Numerical investigation

Analysis of the truss consisting of 12 elements and 9 nodes, as shown in Fig. 2. The coordinates of the truss nodes are given in Table 1. The truss is subjected to the harmonic load: 



**Fig. 2.** Examined truss system

All elements are made of the same material and have the same cross-section area. The shortest element is the sixth element with the length . The amount of the length imperfection in all elements is assumed as a percentage of the length of the sixth element. The analysis is implemented with four accepted values of the amount of length imperfection as following



The data is given as following:



The coefficient in the Newmark method is . Choose the time step Δt=0.0025s, the period of investigation time is 01 second.

The results of the nonlinear dynamic analysis of the truss with four assumed values of the amount of length imperfection are illustrated in Fig. 3-Fig.5

**Table 1.**The coordinates of truss nodes in global coordinate system XOY

|  |  |  |  |
| --- | --- | --- | --- |
| Node | 1 | 2 | 3 |
| Coordinae  (X,Y) cm |  |  |  |
| Node | 4 | 5 | 6 |
| Coordinae  (X,Y) cm |  |  |  |
| Node | 7 | 8 | 9 |
| Coordinae  (X,Y) cm |  |  |  |

Ảnh có chứa biểu đồ

Mô tả được tạo tự động

**Fig. 3.** Displacement - time response (), t = [0, 0.5s]

Ảnh có chứa biểu đồ

Mô tả được tạo tự động

**Fig. 4.** Displacement - time response (), t=[0.5, 1s]

Ảnh có chứa biểu đồ

Mô tả được tạo tự động

**Fig. 5.** Axial load - time response *(N1-t),* t = [0, 1s]

The results in Fig.3-Fig.5 show that when an element's length imperfection is not equal to exactly 1% of the length of the shortest sixth element, the impact on displacement and on axial forces of the truss element is very significant.

The results also show that when an element's length imperfection approaches 0, the solution converges to the case of the component with perfect length.

1. Conclusions

The research shows that the mathematical model based on mixed finite element formula effectively solves the nonlinear dynamic problems of trusses with initial length imperfection.

Applying the mixed finite element formula allows using both displacement and force as unknowns, which not only gives the possibility to put the initial length imperfection of the element into the mixed matrix of truss elements but also simplifies the algorithm in nonlinear dynamic analysis for trusses.

The length imperfection of element significantly impacts on the nonlinear dynamic behaviour of trusses with large displacement. Hence, this parameter should not be neglected in dynamic analysis of trusses.

The length imperfection of the element significantly impacts the nonlinear dynamic behaviour of trusses with large displacement. Hence, this parameter should be accounted in the dynamic analysis of trusses.

The proposed approach and algorithm in the research can be effectively applied to study the nonlinear dynamic behaviour of trusses with length imperfection of elements.

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